

DIRECT CONSISTENCY PROOF OF GENTZEN'S SYSTEM OF
NATURAL DEDUCTION

ANDRÉS R. RAGGIO

Gentzen proves¹ the equivalence of his system of natural deduction with an axiomatization of propositional logic due to Hilbert. This last system is consistent, therefore Gentzen's system also. We avoid this round-about way and give a direct proof that all provable formulas of Gentzen's system of natural deduction are tautologies. From this we can easily infer that the system is consistent.

In the natural deduction we start with assumptions and apply rules of inference. These are the following:

Simple transformation rules

$\frac{A \text{ et } B}{A}$	$\frac{A \text{ et } B}{B}$	"et" elimination
$\frac{A}{A \text{ vel } B}$	$\frac{A}{B \text{ vel } A}$	"vel" introduction
$\frac{\text{abs}}{A}$		"abs" elimination
$\frac{\text{non non } A}{A}$		"non non" elimination

Compound transformation rules

$\frac{A, B}{A \text{ et } B}$	"et" introduction
$\frac{A, A \text{ seq } B}{B}$	"seq" elimination
$\frac{A, \text{non } A}{\text{abs}}$	"abs" introduction

1) Cf. Gentzen, Untersuchungen über das logische Schliessen, *Mathematische Zeitschrift*, vol. 39, p. 417.