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A NEW CONDITION FOR A MODULAR LATTICE

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A lattice L is said to be modular if it satisfies the following axiom:

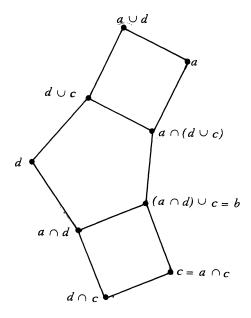
M.
$$[a, b, c]$$
: If $a, b, c \in L$ and $a \ge c$, then $a \cap (b \cup c) = (a \cap b) \cup c$.

Several conditions equivalent to M are known. This paper introduces another characterization of a modular lattice which as far as I know has not been noted.

M'. [a, b, c, d]: If a, b, c, $d \in L$, $a \cap c \leq b$, $a \cap d \leq b$, and c is comparable to a, or c is comparable to d, then $a \cap (c \cup d) \leq b$.

The expression "a is comparable to b" means: $a \le b$ or a > b.

In the finite lattice shown below the elements are represented by dots and x < y if x appears below y and is connected to y by a line segment. This lattice is known to be non-modular and we note that M' does not hold.



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