

A NEW CONDITION FOR A MODULAR LATTICE

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A lattice \mathbf{L} is said to be modular if it satisfies the following axiom:

M . $[a, b, c]$: If $a, b, c \in \mathbf{L}$ and $a \geq c$, then $a \cap (b \cup c) = (a \cap b) \cup c$.

Several conditions equivalent to M are known. This paper introduces another characterization of a modular lattice which as far as I know has not been noted.

M' . $[a, b, c, d]$: If $a, b, c, d \in \mathbf{L}$, $a \cap c \leq b$, $a \cap d \leq b$, and c is comparable to a , or c is comparable to d , then $a \cap (c \cup d) \leq b$.

The expression " a is comparable to b " means: $a \leq b$ or $a > b$.

In the finite lattice shown below the elements are represented by dots and $x < y$ if x appears below y and is connected to y by a line segment. This lattice is known to be non-modular and we note that M' does not hold.

