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ON AN EXTENSION OF A THEOREM OF FRIEDBERG

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In [1], Friedberg showed that any essentially r.e. set (i.e., any r.e. but nonrecursive set) is effectively decomposable into the disjoint union of a pair of essentially r.e. sets.

A careful examination of Friedberg's proof shows that by a slight modification of it, one can specify a method for effectively decomposing an essentially r.e. set ω_u into the union of $\aleph_0$ pairwise-disjoint, essentially r.e. subsets. What is needed, for the modification, is simply to redefine the notion of an index being *satisfied*, so that *satisfaction of e at step a* is defined relative to *the first e + 1* "*a*-parts", P_0^a, P_1^a, . . . , P_e^a, of the components in the decomposition. Minor readjustments in the remainder of the argument then give the desired result; and, indeed, it can be seen that each component is "effectively covered" by the whole set, in a sense we shall shortly define.

However, the bookkeeping details which would attend a formalization of the argument appear formidable; and, therefore, it seems worthwhile to take note of a less technically rococo proof, available in the special case of *creative* sets, of the existence of an infinitary decomposition, with effectively indexed components[1] and with each component "effectively covered" by the whole set. We emphasize, however, that the theorem does also hold for those essentially r.e. sets which do not happen to be creative, though for such sets the more complicated variation on Friedberg's argument, or something like it, seems to be needed.

DEFINITION. *Let* α *be an essentially r.e. set. An* <u>*essentially*</u> *r.e. set* β *is said to be an effective cover of* $\alpha \longleftrightarrow_{df} \alpha \subseteq \beta \ \& \ \overline{\overline{\beta - \alpha}} = \aleph_0 \ \& \sim (\exists \gamma)$ (γ is a recursive set & $\alpha \subset \gamma \subset \beta$).

We proceed now to the theorem which is the object of this note.

THEOREM. *Let* β *be a creative set. Then, there is an effectively enumerable class,* $\Gamma = \{\omega_{f(i)} \mid i \in N\}$ *(here N is the set of all natural numbers, and f a recursive function), such that the* $\omega_{f(i)}$ *are pairwise-disjoint, essentially r.e. sets,* $\beta = \bigcup\Gamma$, *and each* $\omega_{f(i)}$ *is effectively covered by* β.

PROOF. Since β is creative, $\widetilde{\beta}$ has a productive function; and indeed, by results of Dekker ([2, p. 135]) and Myhill ([2, p. 149], [5], and [3, p. 32]),

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