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AN AXIOM-SYSTEM FOR $\{K;N\}$–PROPOSITIONAL CALCULUS RELATED TO SIMONS' AXIOMATIZATION OF S3

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In [4] Simons has shown that Lewis' system S3 can be axiomatized with six mutually independent axiom schemata and the rule of detachment for material implication. As I mentioned in [5], p. 52, it is clear that this formalization of Simons can be reformulated in such a way that instead of axiom schemata the analogous proper axioms

A1 *NMKpNKpp*
A2 *NMKKpqNq*
A3 *NMKKKrpNKqrNKpNq*
A4 *NKNMpNNp*
A5 *NMKpNMp*
A6 *NMKNMKpNqNNMKNMqNNMp*

are adopted together with the following two rules of procedure

I. The rule of substitution ordinarily used in the propositional calculus, but adjusted to the primitive functors "*K*", "*N*" and "*M*".

II. The rule of detachment adjusted to the primitive functors "*K*" and "*N*", viz.:

> *If the formulas "$NK\alpha N\beta$" and "α" are theses of the system, then formula "β" is also a thesis of this system.*

In this note I like to stress a rather interesting fact that the following four theses

B1 *NKpNKpp*
B2 *NKKpqNq*
B3 *NKKKrpNKqrNKpNq*
B4 *NKNKpNqNNKNqNNp*

i.e. the formulas which we can obtain by deleting the modal functor *M* in the axioms *A1*, *A2*, *A3* and *A6* of Simons, taken together with the rules of procedure I and II constitute an axiom-system for the complete classical $\{K; N\}$–

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