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STRUCTURAL RULES OF INFERENCE

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On many occasions the following three rules:

R: $A \vdash A$ (Reflexivity),

E: *If* $A_1, A_2, \ldots, A_n \vdash B$, *then* $A_1, A_2, \ldots, A_n, C \vdash B$ (Expansion),

P: *If* $A_1, A_2, \ldots, A_{i-1}, A_i, A_{i+1}, A_{i+2}, \ldots, A_n, A_{n+1}, A_{n+2} \vdash B$, *then* $A_1, A_2, \ldots, A_{i-1}, A_{i+1}, A_i, A_{i+2}, \ldots, A_n, A_{n+1}, A_{n+2} \vdash B$, *where* $i \leq n + 2$ (Permutation),

are appointed as structural rules of inference for the propositional calculus;[1] on others, **P** and the following generalization of **R**:

GR: $A_1, A_2, \ldots, A_n, A_{n+1} \vdash A_i$, *where* $i \leq n + 1$ (Generalized Reflexivity),

are made to serve in that capacity.[2] I examine here the impact of this switch from **R** and **E** to **GR** upon the proving and deriving of rules of inference for the said calculus.

Let P be a (pure) propositional calculus with '$\sim$' and '$\supset$' as primitive connectives. Let 'A', 'B', and 'C' range in the metalanguage MP of P over the wffs of P. Let (meta)statements of MP of the form '*B is implied in P by* (*or deducible in P from*) $A_1, A_2, \ldots,$ *and* A_n' be abbreviated to read '$A_1, A_2, \ldots, A_n \vdash B$' and called turnstile statements or, for short, T-statements. Let the following four rules serve as intelim rules for '$\sim$' and '$\supset$':

NI: *If* $A_1, A_2, \ldots, A_n, B \vdash C$ *and* $A_1, A_2, \ldots, A_n, B \vdash \sim C$, *then* $A_1, A_2, \ldots, A_n \vdash \sim B$,

NE: *If* $A_1, A_2, \ldots, A_n \vdash \sim \sim B$, *then* $A_1, A_2, \ldots, A_n \vdash B$,

HI: *If* $A_1, A_2, \ldots, A_n, B \vdash C$, *then* $A_1, A_2, \ldots, A_n \vdash B \supset C$,

HE: *If* $A_1, A_2, \ldots, A_n \vdash B$ *and* $A_1, A_2, \ldots, A_n \vdash B \supset C$, *then* $A_1, A_2, \ldots, A_n \vdash C$.

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