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A DECISION PROCEDURE FOR POSITIVE IMPLICATION

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In this paper various formulations of the positive implicational calculus [**PIC**] will be studied. This partial propositional calculus is specified by the axiom schemes (Ax.1) and (Ax.2), given below, and has as a rule of inference, *modus ponens* or detachment. The usual truth-functional tests will not serve as decision procedures for **PIC** because there are pure implicational tautologies that are not theorems of **PIC**. A well known example is Peirce's law

$$((\mathbf{A} \supset \mathbf{B}) \supset \mathbf{A}) \supset \mathbf{A},$$

which, when added to (Ax.1) and (Ax.2), yields classical implication. Gentzen[1] and Wajsberg[2] have obtained decision procedures for **PIC** as corollaries to their decision procedures for the intuitionist propositional calculus. In this paper another decision procedure will be stated and justified. It will be formulated in the implicational fragment of Fitch's method of subordinate proofs [**FI**],[3] a variant of the implicational fragment of Gentzen's system for natural deduction the "Kalkül **LHJ**". The decision procedure offered appears to have two advantages over the other decision procedures: it is in some sense more "natural" because the proof structures in **FI** correspond to the proofs used by ordinary mathematicians; and it is more compact and faster to use (at least by hand). It would appear as if the second advantage would be of interest to those working in mechanical theorem proving, but the author has no information as to whether or not the process of programming the procedure will destroy the advantages it has for hand calculation. Using certain reductions the decision procedure for **PIC** can be extended to include parts of conjunction, disjunction, and negation.[4,5]

The axiom schemes for **PIC** are:

Ax.1. $\mathbf{A} \supset (\mathbf{B} \supset \mathbf{A})$.
Ax.2. $[\mathbf{A} \supset (\mathbf{B} \supset \mathbf{C})] \supset [(\mathbf{A} \supset \mathbf{B}) \supset (\mathbf{A} \supset \mathbf{C})]$.

On the other hand, **FI** has no axioms but only rules, and a notation that allows one to nest proofs within proofs. The rules of **FI** are the following:

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