Notre Dame Journal of Formal Logic
Volume III, Number 3, July 1962

FINITE LIMITATIONS ON DUMMETT'S **LC**

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The propositional system **LC** of [1] can be based on axioms for $\supset$ (implication), $\wedge$ (conjunction), a constant **f**, and definitions for $\vee$ (alternation) and $\urcorner$ (negation), as hereunder. In primitive notation, elementary variables and **f** are wffs, and if α, β are wffs so are $(\alpha \supset \beta)$, $(\alpha \wedge \beta)$. To restore primitive notation in the sequel, replace dots by left parentheses with right terminal mates; in a sequence of wffs separated only by implications, restore parentheses by left association; enclose the whole in parentheses. If S is a system, S_c is its implicational fragment, containing only variables and implications. If α is provable (not provable) in S, we write $\vdash_{S} \alpha$ $(\dashv_{S} \alpha)$; if α is uniformly valued 0 (is not uniformly valued 0) by the matrix $\mathfrak{M}$, we write $\vdash_{\mathfrak{M}} \alpha$ $(\dashv_{\mathfrak{M}} \alpha)$. As a basis for **LC** we take, with detachment and substitution, the axioms and definitions:

1 $\quad p \supset . q \supset p$
2 $\quad p \supset (q \supset r) \supset . p \supset q \supset . p \supset r$
3 $\quad p \supset q \supset r \supset . q \supset p \supset r \supset r$
4 $\quad \mathbf{f} \supset p$
5 $\quad (p \wedge q) \supset p$
6 $\quad (p \wedge q) \supset q$
7 $\quad p \supset . q \supset (p \wedge q)$

Def. $\vee$ $\quad (\alpha \vee \beta) = (\alpha \supset \beta \supset \beta) \wedge (\beta \supset \alpha \supset \alpha)$
Def. $\urcorner$ $\quad \urcorner\alpha = \alpha \supset \mathbf{f}$

[2] shows that 1-3 suffice for $\mathbf{LC}_c$, and it is well known that 1-2 suffice for $\mathbf{IC}_c$, the positive logic. By [1] the infinite adequate matrix for **LC** is $\mathfrak{M} = \langle M, \{0\}, \wedge, \supset, \mathbf{f} \rangle$ where $M = \{0, 1, 2, \ldots, \omega\}$ and

$$a \wedge b = \max(a, b),$$
$$a \supset b = \begin{cases} 0 \text{ if } a \geqq b, \\ b \text{ if } a < b, \end{cases}$$
$$\mathbf{f} = \omega.$$

Received November 14, 1961