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SOME THEOREMS ON THE STRUCTURE OF MUTANT SETS AND THEIR APPLICATIONS TO GROUP AND RING THEORIES

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The author dedicates this paper to the memory of Alan Mathison Turing on the occasion of the 50th anniversary of that logician's birthday, 23rd June, 1912.

§0. Introduction. The purpose of this paper is to present the results of two announcements [1], [2] and to generalize the results of two other papers [3], [4]. The results of §1 are applicable to any general algebraic system, i.e., nonempty set on which there is defined a nonempty index set of closed binary composition laws. For the sake of concreteness some applications to group and ring theories are given in §2 and §3, respectively. Some applications of the general theory to elementary number theory are given elsewhere [5], [6]. Applications in the context of the algebra and logic of relations of abstract mathematical biology are also given elsewhere [4].

§1. General Theory. Recall [6] that a (λ, T)-*mutant* set of an algebraic system $(S, *)$ is a subset M of S that satisfies the condition $M_1 * M_2 * \ldots * M_\lambda \subseteq \overline{M} \cap T$, where $M = M_i$ for all i, λ is an integer ≥ 2 and T together with $*$ forms an algebraic subsystem of $(S, *)$. A (λ, T)-mutant set M of a system $(S, *)$ is said to be a *maximal* (λ, T)-mutant set of $(S, *)$ provided there is no (λ, T)-mutant set of $(S, *)$ which properly contains M.

Theorem 1.1. Every subset of a (λ, T)-mutant set M of $(B, *)$ is a (λ, T)-mutant set of $(B, *)$.

Proof: Put $A = A_i$ and $M = M_i$ for all i. Then $A_1 * \ldots * A_\lambda \subseteq M_1 * \ldots * M_\lambda \subseteq \overline{M} \cap T \subset \overline{A} \cap T$ for every $A \subseteq M$.

Theorem 1.2. Let φ be a homomorphism from $(A, *)$ into $(B, \circ)$. Let M be a (λ, T)-mutant set of $(A, *)$. If $\varphi(\overline{M} \cap T) \subseteq \overline{\varphi(M)} \cap S$ then $\varphi(M)$ is a (λ, S)-mutant set of $(B, \circ)$.

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