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QUANTIFICATION AND Ł-MODALITY

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1. *The Formula* $\Sigma aK\Delta\Theta 1a\nabla\mathbf{L}1a$ *with* Δ *and* ∇ *as Variable Functors.* In his paper "Arithmetic and Modal Logic",[1] Łukasiewicz drew attention to an odd theorem which is deducible when certain arithmetical laws are subjoined to his Ł-modal calculus, namely the theorem (with "Θab" for "$a = b$" and "$\mathbf{L}ab$" for "$a < b$")

$$5.4 \quad \Sigma aK\Delta\Theta 1a\nabla\mathbf{L}1a.$$

What is odd about this theorem is that it holds despite the fact that, according to Łukasiewicz, there exists no positive integer a for which $K\Delta\Theta 1a\nabla\mathbf{L}1a$ is true. But is this really so?

It is noteworthy that while Łukasiewicz's proof of the theorem 5.4 is perfectly rigorous and formal, his proof that there is no positive integer a for which $K\Delta\Theta 1a\nabla\mathbf{L}1a$ holds is not, but depends on the interpretation of Δ and ∇ as *constant four-valued* truth-operators, and on certain truth-value calculations based on this interpretation. If, on the contrary, we interpret Δ and ∇ as *variable two-valued* functors with their range restricted to V and S, with ∇ taking the opposite value to Δ in any given formula,[2] we obtain a different result. For suppose that in the formula 5.4 we assign to Δ the value S and consequently to ∇ the value V. Then if $a > 1$, $K\Delta\Theta 1a\nabla\mathbf{L}1a = KS0V1 = K01 = 0$, but if $a = 1$, $K\Delta\Theta 1a\nabla\mathbf{L}1a = KS1V0 = K11 = 1$; so that with this assignment of values to Δ and ∇, there *is* at least one positive integer, namely 1, for which $K\Delta\Theta 1a\nabla\mathbf{L}1a$ is true. Again, if we assign to Δ the value V and consequently to ∇ the value S, then if $a = 1$, $K\Delta\Theta 1a\nabla\mathbf{L}1a = KV1S0 = K10 = 0$, but if $a > 1$, $K\Delta\Theta 1a\nabla\mathbf{L}1a = KV0S1 = K11 = 1$. Hence for this assignment of values also, there is at least one positive integer, namely any greater than 1, for which $K\Delta\Theta 1a\nabla\mathbf{L}1a$ is true. Hence on both possible assignments of values to Δ and ∇, the formula 5.4 is true in its natural sense, and its appearing as a logical law, i.e. as true for all possible values of its free variables, presents no difficulties.

2. *The Formula* $\Sigma aK\Delta\Theta 1a\ \mathbf{L}1a$ *with* Δ *and* ∇ *as Constant Functors.* It remains true, however, that the Δ and ∇ of the **Ł**-modal system *may* be interpreted, not as above, but as constant four-valued functors; and if they are

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