Notre Dame Journal of Formal Logic
Volume III, Number 3, July 1962

PROGRAMMING THE FUNCTIONS OF FORMAL LOGIC

S. SUMMERSBEE and A. WALTERS

From time to time automatic defices are suggested which will simulate the operations which can be carried out on the truth-tables of formal logic. Specifically, if a formula $F(X_1 \ldots . X_n)$ constructed from the propositional variables $X_1 \ldots . X_n$ and certain logical connectives, is set into the machine it will calculate the truth-value of $F(X_1 \ldots . X_n)$ from the truth-values $x_1 \ldots . x_n$ ($x_i = T$ or F, $1 \leqq i \leq n$) of $X_1 \ldots . X_n$. Probably the best known of such devices is that of W. S. Jevons [1], while one of the most recent is designed to deal with many-valued logic [2].

There is no intrinsic reason for choosing the symbols "T" and "F" to represent the truth-values of a proposition, the symbols "1" and "0" will serve the same purpose. In such an event the truth-value of a formula $F(X_1 \ldots . X_n)$, determined by the truth-values $x_1 \ldots . x_n$ of $X_1 \ldots . X_n$, can be written in the form

$$m = \sum_{j=1}^{j=n} a_i \, 2^{j-1}$$

where $a_i = 0$ or $a_i = 1$. For example, the truth-table for Dpq is

TABLE I

p	q	Dpq
1	1	1
1	0	1
0	1	1
0	0	0

and the final column of the table can be regarded as having either the value 1110 (= 14) or the value 0111 (= 7) depending on the convention adopted. Alternatively, the values 0, 3, 5, 7 for m will give the truth-value of Dpq for given values of p and q; for example 5 would be rewritten as 101 and hence for $p = 1$ and $q = 0$, $Dpq = 1$.

Received September 17, 1961