# A NOTE ON A WEAKENED GOLDBACH-LIKE CONJECTURE

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*Introduction.* This paper presents a constructive existence proof for a certain maximal doubly mutant set [1] in the algebraic system determined by the additive and multiplicative monoids of all nonnegative integers. It shows that if a weakened version of Goldbach's conjecture is a theorem then that constructed maximal property can be extended to be relative to *all* nonnegative integers.

1. *Results.*

*Lemma:* In the additive and multiplicative monoids of nonnegative integers there exists, *in a constructive sense*, a doubly mutant set that is a maximal one relative to the set of all positive odd integers.

*Proof:* Put $E_0$ equal to the set of all odd prime numbers, i.e., $E_0 = \{3, 5, \ldots\}$. Put $E_1 = \{P_{i_1} \cdot P_{i_2} \cdot P_{i_3} : P_{i_j} \in E_0\}, \ldots, E_n = \{P_{i_1} \cdot P_{i_2} \cdots P_{i_{2n+1}} : P_{i_j} \in E_0\}, \ldots$. Put $S = \bigcup_{i=0}^{\infty} E_i$. Clearly $S$ is a doubly mutant set. Specifically $S$ is an additive mutant set since, e.g., the sum of two odd integers is an even integer. It is a multiplicative mutant as is easily verified by using the fact that the set of all positive odd integers is an additive mutant and by using the *Fundamental Theorem of Arithmetic*. Clearly $S$ is a doubly mutant set that is a maximal one relative to the set of all odd positive integers since by the *Fundamental Theorem of Arithmetic* $S \cdot S$ equals the complement of $S$ relative to the set of all positive odd integers greater than the multiplicative idempotent element *1*, which cannot be an element of a multiplicative mutant set.

Now suppose that $S + S$ equals the set of all positive even integers greater than four (a weakened form of Goldbach's conjecture [2], [3]). Then $S$ is a doubly mutant set that is a maximal one relative to the set of *all* nonnegative integers and, *a fortiori*, if Goldbach's conjecture is a theorem, then, among numerous other things, $S$ is a doubly mutant set that is a maximal one relative to the set of all nonnegative integers.

## REFERENCES

[1] A. A. Mullin, "On Multiply Mutant Sets," *Notices Amer. Math. Soc.*, vol. 8, No. 4, August, 1961, p. 357.