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A NOTE ON THE REGULAR AND IRREGULAR MODAL SYSTEMS OF LEWIS

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I say that a modal formula α is regular, if after deleting the modal functors L and M,[1] if they occur in α, and after replacing the modal functors for more then one argument, as e.g. $\mathfrak{C}$ and $\mathfrak{E}$, if they occur in α, by the corresponding functors from the classical propositional calculus, throughout α, this formula becomes a thesis of the bi-valued propositional calculus. On the other hand, if after such operations α is transformed into a meaningful propositional formula, but not into a thesis, then α is called an irregular modal formula. Thus, e.g., $\mathfrak{C}Lpp$ is a regular modal formula, but Lewis' C13: MMp[2] is irregular. Correspondingly, the modal systems in which no irregular formula occurs are called regular. And, obviously, the irregular modal systems are such that they contain the irregular theses. Thus, e.g., the systems S1 - S5 and T are regular, but the system S6 of Lewis is irregular.

In this note I shall prove that any Lewis' modal system which contains system T of Feys-von Wright[3] must be regular. On the other hand, it will be shown that there are systems in which the rule:

RI *If α is provable in the system, then also $L\alpha$ is provable in the system.*

holds, and which have irregular, quasi-normal (in the sense of Scroggs) extensions.[4]

1. *System T°*. It is known[5] that an addition of **RI** as a new rule of procedure to S1 of Lewis gives a system inferentially equivalent to system T. In [11] Yonemitzu has proved that an addition to S1 of an arbitrary formula which has the form $LL\alpha$ and is such that $L\alpha$ is a thesis of S1, generates rule **RI** and, therefore, gives a system inferentially equivalent to T.

It can be proved easily[6] that an addition to S1° of an arbitrary formula of the form $LL\alpha$ and such that $L\alpha$ is a thesis of Feys' system S1°[7] as a new axiom constitutes a system, called T°, in which rule **RI** is also provable. Group I of Lewis-Langford[8] shows that formula $LL\alpha$ which satisfies the, above mentioned, condition is independent from the system S1°. On the

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