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ON THE CONNECTION OF THE FIRST-ORDER FUNCTIONAL CALCULUS WITH MANY-VALUED PROPOSITIONAL CALCULI

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From the results presented in my paper [2] it follows that it is possible to approximate the first-order functional calculus by many valued propositional calculi; in this paper* we shall describe this approximation.

We shall use the terminology of [2] and in particular:

(1) individual variables: $x_1, x_2, \ldots$ [or simply x],

(2) apparent individual variables: $a_1, a_2, \ldots$ [or simply a],

(3) finite number of functional variables: $f_1, \ldots, f_c$,

(4) logical constants: ' (negation), + (alternative), Π (general quantifier),

(5) atomic expressions: $R, R_1, R_2, \ldots$; expressions: $E, F, G, E_1, F_1, G_1, \ldots$[1]

(6) $w(E)$ –the number of different individual [$p(E)$–apparent] variables occurring in the expression E,

(7) $\{i_m\}$ –the sequence $i_1, \ldots, i_m$; $\{i_{w(E)}\}$–all different indices of those and only those individual variables which occur in E,

(8) $n(E) = \max\{w(E) + p(E), \max\{i_{w(E)}\}\}$,

(9) $\bar{n}(E) = n(E)$, if E is an alternative of normal forms, $\bar{n}(E) = \max\{n(E), n(F)\}$, where F is the simplest alternative of normal forms equivalent to E, in the opposite case (we choose an arbitrary alternative),

(10) $\bar{c}$ –maximum of arguments of $f_1, \ldots, f_c$,

(11) $E(u/z)$ –the expression resulting from E by substitution of u for each occurrence of z in E (with usual conditions),

(12) $C(E)$ –the set of all significant parts of the formula E: $H \epsilon C(E)$.[2] $\equiv$. $H = E$ or there exist F, G, H_1 such that: $(H = F) \wedge (E = F') \vee \{(H = F) \vee (H = G)\}$ $(E = F + G) \vee (\exists i)\{H = H_1(x_i/a)\} \wedge (E = \Pi a H_1)$,

(13) Skt –the set of all formulas of the form $\Sigma a_1 \ldots \Sigma a_i \Pi a_{i+1} \ldots \Pi a_k F$, where F is a quantifierless expression containing no free variables, Πa_j is the sign of the universal quantifier binding the variable a_j and $\Sigma a_j G = (\Pi a_j G')'$, $j = 1, \ldots, k$.[3]

*An abstract of this paper appeared in [5].

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