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INTUITIONISM RECONSIDERED

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It has long been known that standard Gentzen rules of inference for PC_I, the intuitionist propositional calculus, will do for PC_C, the classical propositional calculus, once the intuitionist elimination rule for '$\sim$', namely:

NE_I: $A, \sim A \vdash B$,

is strengthened to read:

NE_C: $\sim \sim A \vdash A$.

It has recently been shown that the said rules will also do for PC_C once the intuitionist elimination rule for '$\supset$', namely:

HE_I: $A, A \supset B \vdash B$,

is strengthened to read:

HE_C: $A \supset B, (A \supset C) \supset A \vdash B$.[1]

We shall now show that the said rules will finally do for PC_C once the intuitionist elimination rule for '$\equiv$', namely:

BE_I: (a) $A, A \equiv B \vdash B$
(b) $A, B \equiv A \vdash B$,

is strengthened to read:

BE_C: (a) $A, (C \equiv A) \equiv (C \equiv B) \vdash B$
(b) $A, (C \equiv B) \equiv (C \equiv A) \vdash B$.

The debate between intuitionist logic and classical logic, a debate which originally centered around '$\sim$' and has more recently come to center around '$\supset$', can thus be made to center around '$\equiv$' as well. Details are as follows.

Let all five of '$\supset$', '$\sim$', '&', 'v', and '$\equiv$' serve as primitive connectives of PC_C; let 'A', 'B', and 'C' range over the well-formed formulas of PC_C; let '$A_1, A_2, \ldots, A_n \vdash B$', where $n > 0$, be short for 'B is deducible in PC_C

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