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FOUR TYPES OF GENERAL RECURSIVE WELL-ORDERINGS

SHIH-CHAO LIU

In this paper the term 'g.r. (general recursive) well-ordering' refers to g.r. well-orderings of the set of all the natural numbers. Markwald in [1, Satz 5] gave an example showing that some of his recursively enumerable well-orderings can exhibit non-constructive character in an important aspect. In this paper we shall give more examples of similar kind; they are all g.r. well-orderings and are more or less non-constructive.

The examples suggest a classification of g.r. well-orderings into four types, which is based on the following three conditions:

α) There are two g.r. functions $\mathbf{H}(n)$ and $\mathbf{G}(n)$ such that $\mathbf{H}(n) = 0$, 1 or 2 according as n is the first element, a successor or a limit and $\mathbf{G}(n) = 0$ or 1 according as n is the last element or not.

β) There is an effective method for finding the successor of any element which is not the last one.

γ) There is an effective method for finding the limit of any g.r. increasing bounded sequence $\{a_j\}$.

Here the term 'effective' is used in the sense that, given a set of entities, each of which is associated with a unique number in some specified manner, then a method for finding the associated number for each such entity is effective if, for any effectively given sequence of such entities $\{E_j\}$, the number associated with each E_j can be found by that method and the number found is a g.r. function of j.

The four types of g.r. well-orderings are characterized by the conditions as shown in the following.

Types of g.r. well-ordering	I	II	III	IV
Conditions satisfied		α	α, β	α, β, γ
Conditions not satisfied	α, β, γ	β, γ	γ	

In view of the nature of the conditions α, β, γ, g.r. well-orderings of each of the types II, III, IV can be considered as 'more constructive' than

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