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A CONTRIBUTION TO THE AXIOMATIZATION OF LEWIS' SYSTEM S5

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In [7] Simons has shown that the following six axiom schemata:[1]

H1 $\vdash [\alpha \prec (\alpha \wedge \alpha)]$.
H2 $\vdash [(\alpha \wedge \beta) \prec \beta]$.
H3 $\vdash \{[(\gamma \wedge \alpha) \wedge \sim (\beta \wedge \gamma)] \prec (\alpha \wedge \sim \beta)\}$.
H4 $\vdash (\sim \Diamond \alpha \rightarrow \sim \alpha)$.
H5 $\vdash (\alpha \prec \Diamond \alpha)$.
H6 $\vdash [(\alpha \prec \beta) \prec (\sim \Diamond \beta \prec \sim \Diamond \alpha)]$.

(in which "$\alpha \rightarrow \beta$" and "$\alpha \prec \beta$" are used as the abbreviations of "$\sim (\alpha \wedge \sim \beta)$" and "$\sim \Diamond (\alpha \wedge \sim \beta)$" respectively) together with the rule of inference:

if α *is provable and* $(\alpha \rightarrow \beta)$ *is provable, then* β *is provable,*

constitute a modal system inferentially equivalent to Lewis' system S3. Moreover, he also proved that by adding the schematic analogue of Lewis' C 10.1, viz.

H7 $\vdash (\Diamond \Diamond \alpha \prec \Diamond \alpha)$

to *H1* - *H6* we obtain an axiomatization inferentially equivalent to S4, and that the axiom schemata *H1* - *H7* are mutually independent. On the other hand he remarked that although, obviously, one can get an axiomatization of S5 by adding to *H1* - *H6* the schematic analogue of C 11, viz.

H8 $\vdash (\Diamond \alpha \prec \sim \Diamond \sim \Diamond \alpha)$

he was unable to prove the mutual independence of *H1* - *H6* and *H8*. In [1] Anderson has shown that an addition of the following axiom schema

S $\vdash [(\sim \Diamond \sim \alpha \prec \Diamond \alpha) \prec (\alpha \prec \sim \Diamond \sim \Diamond \alpha)]$

to Simons' *H1* - *H6* gives a set of mutually independent axiom schemata for S5.

In this paper I shall show that:

1) the Simons' formulas *H1*, *H2*, *H3*, *H4*, *H6* and *H8* imply *H5*.

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