THREE SET–THEORETICAL FORMULAS

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The set–theoretical formula which says that:

$\mathfrak{A}$. *If* $\mathfrak{m}$ *is a cardinal number which is not finite, then there exists no cardinal number* $\mathfrak{n}$ *such that* $\mathfrak{m} < \mathfrak{n} < 2^{\mathfrak{m}}$,

is called the generalized continuum hypothesis. It is known[1] that $\mathfrak{A}$ is inferentially equivalent to:

$\mathfrak{B}$. *The axiom of choice*

taken in conjunction with

$\mathfrak{C}$. *Cantor's hypothesis on alephs*

which says that

For any ordinal number α: $2^{\aleph_\alpha} = \aleph_{\alpha+1}$

Moreover, it is known[2] that $\mathfrak{C}$ is inferentially equivalent to:

$\mathfrak{D}$. If $\mathfrak{a}$ is an arbitrary aleph, then there exists no cardinal number such that $\mathfrak{a} < \mathfrak{n} < 2^{\mathfrak{a}}$.

The aim of this note is to show that the following three set–theoretical formulas:

A. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{n} < 2^{\mathfrak{m}}$, *then* $\mathfrak{n} \leqslant \mathfrak{m}$.[3]

B. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{n} < 2^{\mathfrak{m}}$, *then either* $\mathfrak{n} \leqslant \mathfrak{m}$ *or* $\mathfrak{m} < \mathfrak{n}$.

C. *For any cardinal number* $\mathfrak{n}$ *which is not finite and any cardinal number* $\mathfrak{a}$, *if* $\mathfrak{a}$ *is an aleph and* $\mathfrak{n} < 2^{\mathfrak{a}}$, *then* $\mathfrak{n} \leqslant \mathfrak{a}$.

are such that formula **A** is equivalent to $\mathfrak{A}$, formula **B**–to $\mathfrak{B}$ and formula **C**–to $\mathfrak{C}$. It seems to me that this fact, which as far as I know has not been noticed, is of some interest, because the formulas **A**, **B**, and **C** having very similar structure elucidate the mutual connections among the fundamental laws $\mathfrak{A}$, $\mathfrak{B}$ and $\mathfrak{C}$.

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