ON A FAMILY OF PARADOXES

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1. Some paradoxical statements are, on the face of it, awkward for the propounder only, while some are also awkward for the looker-on. The Eubulidean version of the Liar paradox is of the second sort—if a man says 'What I am now saying is false', not only he himself but we who look on seem forced to say contradictory things (that his statement must be true because even if it were false it would be true, *and* that it must be false because even if it were true it would be false). On the other hand, if Epimenides the Cretan says that nothing said by a Cretan is the case, it appears that he has landed *himself* in a hole, but the beholder can contemplate his position without unease, simply saying that what Epimenides says must be false because even if it were true it would be false, and so concluding that it *is* false without further ado.

2. Church, however, has pointed out that there is a *little* further ado for the beholder nevertheless. For if what Epimenides says is false, then its contradictory, i.e. that *something* said by a Cretan is the case, must be true, and as the only Cretan statement we have been told about is false, this true Cretan statement which there must be, must be some other one than this. In other words, this one Cretan statement cannot even be made unless some other Cretan statement is made also.

3. Let us try formalising this proof in the propositional calculus enriched by (*a*) variables standing for monadic proposition-forming 'functors' of propositions (we shall use the one variable '*d*' for this purpose), and (*b*) quantifiers binding variables of any categories. We shall use *U* for the universal quantifier and *E* for the existential; for the rest Łukasiewicz's symbols, with *Q* for material equivalence as in *Aristotle's Syllogistic*. For postulates: substitution for variables (with the usual restrictions in the presence of quantifiers) detachment, Łukasiewicz's rules for the quantifiers, definitions of the various truth functions in terms of *C* and *U* ($Np = CpUpp$), and the one axiom *CCCpqrCCrpCsp*. This gives the full ordinary propositional calculus, but does *not* give any laws like *CdpCdNpdq*, *CQpqCdpdq*, *CddUppdp*, which in effect restrict the values of *d* to truth-functors. *d* can thus be used to stand for, among other things, the functor 'It is said by a Cretan that—', and where it occurs in the proofs below as a

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