ON A RECENT ALLOTMENT OF PROBABILITIES TO OPEN AND CLOSED SENTENCES

HUGUES LEBLANC

Probabilities, though frequently allotted to closed sentences, have rarely been allotted to open ones. The recent scheme by Kemeny, Mirkil, Snell, and Thompson in *Finite Mathematical Structures* for allotting probabilities to sentences of the form '$f(x) = a$' is therefore of considerable interest.[1] It has, however, a shortcoming which I should like to discuss here and, possibly, remedy.

Let 'f' be a functional constant, 'x' an individual variable, and 'a' an individual constant; let U be the (finite) set of values of 'x'; and let A be the subset of U whose members satisfy '$f(x) = a$'. Kemeny *et al.* then take the probability of '$f(x) = a$' to be $m(A)$, where $m(A)$ is the measure (in some appropriate sense of the word 'measure') of A.[2] Their scheme is attractive enough and mirrors to some extent what mathematicians understand by the probability of a set.[3] Kemeny *et al.* are careful, of course, to restrict it to open sentences of the form '$f(x) = a$'. Consider, however, a closed sentence of the kindred form '$f(b) = a$', where 'b' is an individual constant. Since '$f(b) = a$' does not contain any occurrence of 'x', it would normally be held to be satisfied by every member of U when true, by none when false. One would accordingly expect Kemeny *et al.* to take the probability of '$f(b) = a$' to be *1* when '$f(b) = a$' is true, *0* when '$f(b) = a$' is false. Yet in their scheme for allotting probabilities to closed sentences, a scheme I shall go into below, they let the probability of a closed sentence equal *1* only when the sentence is logically true, *0* only when it is logically false.[4] '$f(b) = a$' being neither logically true nor logically false, its probability must therefore differ by that scheme from either one of *1* and *0*, a disturbing enough result.

The difficulty becomes even more acute when the calculus, call it C, to whose sentences probabilities are allotted is a simple applied predicate calculus of the first order with identity.

Assume indeed that a set D of individuals has been singled out as the domain of C, a member of D paired with each individual constant W of C as the individual designated by W, and a class of ordered n-tuples of members of D paired with each n-adic predicate constant F of C as the extension of

Received September 20, 1960