A FORMAL SYSTEM

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The present paper arose in connection with my teaching of projective geometry. In trying to get away from too much reliance on drawings I used the notation (ABC) to indicate that the three points A, B, C are collinear; it is possible to express the hypotheses and the conclusions of all the theorems in terms of collinearity; it then appeared possible to dispense not only with drawings but also with other aspects of intuition, and I became interested in how far one can go in that direction. I have not found in literature a formal system that fitted my requirements, so I attempted to devise my own. This attempt is described in what follows. Its origin in connection with projective geometry influences the exposition but it is hoped that the point of view is applicable to more general situations.

In discussing formal systems it is possible roughly to distinguish four stages in the development: in the first stage, we deal with objects; in the second stage, having given names to the objects, we lead a discussion using these names; in the third stage we still handle these names but we have achieved a certain degree of abstraction, the names, or symbols apply to generalizations; they still stand for objects but there may be different sets of objects to which the same names apply and what we say applies to all these sets. In the fourth stage we do not consider that the signs or symbols stand for something - we just operate on them according to certain rules (in a way, we deal with symbols as objects so that the fourth stage resembles the first).

There is importance that can be attached to activity in any of these stages, and it also in important to investigate the transitions from one stage to another but in this essay we are interested principally in the fourth stage.

In order to emphasize the formal character of the system we shall speak of *letters* rather than symbols the word symbol having the connotation that it stands for something, that it symbolizes some object. Further, we will not speak of sentences or propositions; instead of that we'll speak of *rows* (of letters), and instead of theory we'll speak of a *page*. Corresponding to hypotheses and conclusions of a proposition we'll speak of the left side and the right side of a row; we'll separate them by the letter ⊃. We are not