# A SIMPLE FORMULA EQUIVALENT TO THE AXIOM OF CHOICE

BOLESŁAW SOBOCIŃSKI

It is well known that a theorem:

I. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$, *if* $\mathfrak{m} < \mathfrak{n}$, *then there exists a cardinal number* $\mathfrak{p}$ $(>0)$ *such that* $\mathfrak{n} = \mathfrak{m} + \mathfrak{p}$.

is provable in the set theory (and also in logic) without the aid of the axiom of choice[1]. It can be shown easily that a modification of this theorem, viz.

$\mathrm{I}^{\circ}$. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{m} < \mathfrak{n}$, *then* $\mathfrak{n} = \mathfrak{m} + \mathfrak{n}$.

is inferentially equivalent to an assumption:

$\mathfrak{A}$. *For any cardinal number* $\mathfrak{m}$ *which is not finite:* $2\mathfrak{m} = \mathfrak{m}$.

This equivalence can be proved e.g. by an elementary application of a known theorem of F. Bernstein, viz.

$\mathfrak{B}$. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$, *if* $2\mathfrak{m} = 2\mathfrak{n}$, *then* $\mathfrak{m} = \mathfrak{n}$.

which is provable without the aid of the axiom of choice[2].

As far as I know it has not been observed yet that a formula analogous to I but formulated for the multiplication of cardinals:

II. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{m} < \mathfrak{n}$, *then there exists a cardinal number* $\mathfrak{p}$ $(>0)$ *such that* $\mathfrak{n} = \mathfrak{m}\mathfrak{p}$.

is inferentially equivalent to the axiom of choice. From a proof which is presented below of this equivalence it follows obviously that a formula analogous to $\mathrm{I}^{\circ}$, viz.

$\mathrm{II}^{\circ}$. *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{m} < \mathfrak{n}$, then $\mathfrak{n} = \mathfrak{m}\mathfrak{n}$.

possesses also the same property.

*Proof:* In order to show the discussed equivalence it is sufficient to prove that the axiom of choice is a consequence of the formula II, as it is evident that II follows from that axiom. Having the formula II we can establish the following:

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