STUDIES IN THE AXIOMATIC FOUNDATIONS OF BOOLEAN ALGEBRA

CZESŁAW LEJEWSKI

Section III

In the preceding sections we have shown that a deductive system, to be referred to as System 𝔄, can be constructed with *A1* as the only axiom and with R1 – R5 as the rules of inference. This system is strong enough to yield ordinary systems of Boolean Algebra. In contradistinction to such systems we can describe System 𝔄 as a system of Boolean Algebra with definitions since it is the rules of definition, R4, and R5 in particular, that are the distinguishing characteristic of the system.

In the present Section two other systems of Boolean Algebra with definitions are outlined and shown to be inferentially equivalent to System 𝔄. The one, to be known as System 𝔅, is based on the functor of strong inclusion as the only undefined term. Its only axiom takes the form of the following thesis:

B1. $[a\,b] :: a \sqsubset b \,.\equiv .\cdot. [\exists\, c] \,.\, c \sqsubset a \,.\cdot. [c\,d] : c \sqsubset d \,.\, c \sqsubset a \,.\supset . [\exists\, e] \,.\, e \sqsubset d \,.\, e \sqsubset b$

As its rules of inference we have R1 – R4, and instead of R5 we have BR5, which allows us to add to the system new theses of the form

XII $[a \ldots] :: a \sqsubset x \,.\equiv .\cdot. a \sqsubset a \,.\cdot. [b] : b \sqsubset a \,.\supset . [\exists\, c] \,.\, c \sqsubset b$
$\phi(c)$

provided they satisfy certain conditions analogous to those postulated by R5.

The other system, which we shall call System ℭ, makes use of the functor of partial inclusion as the only undefined term.

Thesis

C1. $[a\,b] :: a \Delta b \,.\equiv .\cdot. [\exists\, c] .\cdot. c \Delta a .\cdot. [d] : c \Delta d \,.\supset . a \Delta d \,.\, b \Delta d$

serves as the only axiom of the system, whose rules of inference are the same as those of System 𝔄 except that instead of R5 we have CR5.

Received February 19, 1960.