

A PARADOX REGAINED<sup>1</sup>

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Another attempt has recently been made (by R. Shaw) to analyze a puzzle variously known as the Hangman,<sup>2</sup> the Class A Blackout,<sup>3</sup> the Unexpected Egg,<sup>4</sup> the Surprise Quiz, the Senior Sneak Week,<sup>5</sup> the Prediction Paradox,<sup>6</sup> and the Unexpected Examination.<sup>7</sup> The following simple version of the paradox is sufficient to exhibit the essential features of all other versions. A judge decrees on Sunday that a prisoner shall be hanged on noon of the following Monday, Tuesday, or Wednesday, that he shall not be hanged more than once, and that he shall not know until the morning of the hanging the day on which it will occur. By familiar arguments it appears both that the decree cannot be fulfilled and that it can.

Treatments of the paradox have for the most part proceeded by explaining it away, that is, by offering formulations which can be shown not to be paradoxical. We feel, with Shaw, that the interesting problem in this domain is of a quite different character; it is to discover an exact formulation of the puzzle which is genuinely paradoxical. The Hangman might then take a place beside the Liar and the Richard paradox, and, not unthinkably, lead like them to important technical progress.

Before the appearance of Shaw's article, we had considered a form of the paradox essentially identical with his, and found it, contrary to his assertion, not to be paradoxical. At the same time we were successful in obtaining several versions which are indeed paradoxical. The present note is intended to report these observations.

It is perhaps advisable to begin with a simple treatment due to Quine. The judge's decree,  $D_1$ , delivered Sunday, is that one of the following three conditions will be fulfilled: (1) The prisoner  $K$  is hanged on Monday noon, but not on Tuesday or Wednesday noon, and  $K$  does not know on Sunday afternoon that ' $K$  is hanged on Monday noon' is true; (2)  $K$  is hanged on Tuesday noon, but not on Monday or Wednesday noon, and  $K$  does not know on Monday afternoon that ' $K$  is hanged on Tuesday noon' is true; or (3)  $K$  is hanged on Wednesday noon, but not on Monday or Tuesday noon, and  $K$  does not know on Tuesday afternoon that ' $K$  is hanged on Wednesday noon' is true.

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