

ON THE SINGLE AXIOMS OF PROTOTHETIC

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In this paper I should like to present the results of my unpublished investigations ¹⁾ concerning axiom systems of protothetic. Strictly speaking, only a system of protothetic called \mathfrak{S}_5 , will be considered here. It seems to me that this investigation may interest students of propositional calculus and the related subjects, since the deductions which will be used, sometimes unexpected and rather difficult, not only explain in some degree the structure of protothetic, but can also throw light upon several problems connected with various systems of propositional calculus. Because, generally, protothetic is still a little known theory, at the beginning I have to give several, possibly short, explanations concerning it. Without them the subject of this paper and the proofs presented below would hardly be understandable for the reader. Thus, in the first chapter a short description of protothetic and the necessary information about the rules of procedure of the system \mathfrak{S}_5 will be given. There will also be added some history of the researches concerning the single axioms of protothetic and related problems. Especially, I shall discuss here briefly the metatheorems \mathfrak{L} (of Leśniewski) and the stronger \mathfrak{S} (mine). In the second chapter I shall present a combined proof: 1) that my axiom A_n can serve as a single (and probably the shortest) axiom of the system \mathfrak{S}_5 of protothetic, and 2) that the above mentioned metatheorem \mathfrak{S} is sufficient to check the completeness of any axiom system of protothetic. In the third and the last chapter it will be shown in the shortest possible way how the classical propositional calculus and the quantification theory for protothetical formulas can be obtained in the field of the system \mathfrak{S}_5 .

Instead of an authentic symbolism of Leśniewski²⁾ introduced by him mostly in order to formulate the rules of procedure in the most precise way I shall use here a more convenient Peano-Russell symbolism modified in such a manner that it will become adjusted to the requirements of protothetic. Any one learned in logic will understand these modifications without difficulty. Only, in order to avoid possible misunderstandings I would like to note: 1) that the parentheses of the form "[]" will be used here exclusively as the left and right scopes of the general quantifier, and 2) that if in a formula a quantifier is immediately preceded and closed by an even collection of