INDEPENDENCE OF FARIS-REJECTION-AXIOMS

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[1] questions the independence of the rejection-axioms in [2]. This system for non-void classes, based on the primitive expressions: 1xy (x and y are co-extensive), 2xy (x is properly included in y), 3xy (x and y include a common subclass and each a distinct subclass), 5xy (x and y have no common subclass), was shown equivalent to the syllogistic of [3] in [4] where some alternative assertion-axioms were given. The non-independence of the original set of assertion-axioms is proved in [5]. The resulting, independent set, with original numbering, is:

1. $1aa$ 3. $C1abC3cb3ac$ 4. $C1abC2bc2ac$ 5. $C1abC5cb5ac$
6. $C2abC2bc2ac$ 7. $C2abC5bc5ac$ 8. $CN1abCN2abCN3abCN2ba5ab$
9. $C1abKN2abKN3abN5ab$ 10. $C3abKN2abN5ab$

The rejection-axioms, which will here be proved independent, are:

51. $C2abN2bc$ 52. $C2abN5bc$ 53. $C2abC3bcN2ac$
54. $C2abC3bcN3ac$ 55. $C2abC3bcN5ac$ 56. $C2abC2cbN5ac$
57. $C3abC2bcN2ac$ 58. $C3abC3bcN3ac$ 59. $C3abC3bcN5ac$
60. $C3abC5bcN5ac$ 61. $C5abC5bcN5ac$

Besides the basic rules of rejection usual for such systems, viz. from $\dashv$Y and $\dashv$CXY to infer $\dashv$X, and, from $\dashv$Y, to infer $\dashv$X when Y is a substitution in X, there is a special rule (RG), discussion of which is reserved till later.

The method adopted is to transfer $\dashv -n$ from the rejection- to the assertion-axioms and find an interpretation which (always) verifies the newly augmented assertion axioms and (sometimes) falsifies the remaining rejects. In every case we shall use a subdomain of the general domain for which the system is intended, thus ensuring continued verification of the original assertion-axioms and applicability of the rules. In Tables I and II below, each capital letter represents a class exclusive of all the others, juxtaposition expressing the logical sum. For each $\dashv -n$ transferred to the assertion axioms we use one or other of the tables less line n, and the domain of interpretation is precisely the other classes that thus come to be tabled. Table I is used for $\dashv$51, $\dashv$53 – $\dashv$59; Table II for $\dashv$52, $\dashv$60 and $\dashv$61. In each table

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