

STUDIES IN THE AXIOMATIC
FOUNDATIONS OF BOOLEAN ALGEBRA

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Introduction

One way of characterizing Boolean Algebra would be to say that it consists of all those theses which can be deduced from the following axiom-system due to Schröder:

- S1. $[a]. a \subset a$
 S2. $[abc] : a \subset b. b \subset c. \supset. a \subset c$
 S3. $[a]. \wedge \subset a$
 S4. $[a]. a \subset \vee$
 S5. $[abc] : c \subset a \cap b. \equiv. c \subset a. c \subset b$
 S6. $[abc] : a \cup b \subset c. \equiv. a \subset c. b \subset c$
 S7. $[abc]. a \cap (b \cup c) \subset (a \cap b) \cup (a \cap c)$
 S8. $[a]. a \cap \sim(a) \subset \wedge$
 S9. $[a]. \vee \subset a \cup \sim(a)'$

The totality of theses as deduced from S1 – S9 will be referred to as System \mathfrak{S} .

As is well known, Boolean Algebra lends itself to various interpretations. For the purpose of the present enquiry I propose to adhere to what may be styled as the *ontological* interpretation. It is important to realize as clearly as possible what this interpretation presupposes and what it implies.

Let us begin with a few introductory remarks on names and name-like expressions in general. Names and name-like expressions of ordinary language can be divided into two classes: the class of referential names, which subdivide into unshared names and shared names, and the class of non-referential or fictitious names.²⁾ A referential name names or designates at least one object. A fictitious name behaves, as regards its syntax, like a referential one but it fails to name or designate anything at all. In accordance with this classification names or name-like expressions such as 'Socrates', 'the husband of Xanthippe', 'philosopher', 'inhabitant of London', are all referential names, whereas 'Pegasus', 'mermaid', 'object which does not exist,' etc., are examples of non-referential names.

Now, the ontological interpretation of Boolean Algebra demands that the variables, 'a', 'b', 'c', etc., should be regarded as *naminal* variables, i.e., as variables for which names, referential or non-referential, could be substituted.

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