INVESTIGATIONS ON A COMPREHENSION AXIOM WITHOUT NEGATION IN THE DEFINING PROPOSITIONAL FUNCTIONS

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Introduction

In the paper "Bemerkungen zum Komprehensionsaxiom" in *Zeitschr. f. math. Logik und Grundl.d. Math.*, Bd 3 (1957), p. 1-17, I showed that antinomies of the same kind as Russell's could be avoided in set theory, if this was based on a certain logic, due to Łukasiewicz, with infinitely many truth values. Indeed I proved the existence of domains such that the axiom of comprehension was satisfied for elementary propositional functions ϕ, that is ϕ being built from atomic propositions $u \epsilon v$ by use of conjunction, disjunction, implication and negation only. Later I proved the same for a certain 3-valued logic as shown in a paper which will appear in *Math. Scand.* Here I shall show in §1 and §2 that the same is true even for ordinary 2-valued logic, provided that only conjunction and disjunction are allowed in ϕ. In §3 I prove that also the axiom of extensionality is valid for the domains constructed in §1 and §2. I call the ϕ constructed in this way positive propositions, abbreviated p. pr. The words "atomic propositions" are abbreviated to at. pr.

The two truth values can be 0 (false) and 1 (true). In the sequel I write the conjunction of A and B as $A \wedge B$ and their disjunction as $A \vee B$. Further $A(x)$ for all x is written $\bigwedge xA(x)$ and $A(x)$ for some x is written $\bigvee xA(x)$.

Now the p. pr. can be defined inductively as follows.

1. The truth constants 0 and 1 are p. pr.
2. Every at. pr. $x \epsilon y$ is a p. pr. Here x and y are free variables.
3. If A and B are p. pr., so are $A \wedge B$ and $A \vee B$. The latter have the free and bound variables occurring in A and B.
4. If $A(x, x_1, \ldots, x_n)$ is a p.pr. with $x, x_1, \ldots, x_n$ as free variables $\bigwedge$ x A $(x, x_1, \ldots, x_u)$ and $\bigvee$ x A $(x, x_1, \ldots, x_n)$ are p. pr. with x as bound variable, $x_1, \ldots, x_n$ still as free variables, while the eventually occurring bound variables in $A(x, x_1, \ldots, x_n)$ remain bound in the latter expressions.

   If a set y is such that

   $$\bigwedge x \quad ((x \epsilon y) = U(x, x_1, \ldots, x_n))$$

   is true, where $x, x_1, \ldots, x_n$ are the set variables in the p.pr. U, then y is a set function of $x_1, \ldots x_n$.

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