Notre Dame Journal of Formal Logic
Volume VIII, Numbers 1 and 2, April 1967

THREE-VALUED PROPOSITIONAL FRAGMENTS WITH CLASSICAL IMPLICATION

IVO THOMAS

In [1] V. Vučković discussed a generalized system of recursive arithmetic, for which also see [2], in which he found he could obtain the representing equations of a three-valued propositional logic containing classical implication, a weak negation and two systems of conjunction-alternation. He suggested a third system as the union of these two, retaining the weak negation, in fact the system A discussed in [3], but later realised that the model of such a union was unobtainable in the arithmetic. We show that any complete axioms for his matrices

C	0	1	2	N_1	N_2
$*0$	0	1	2	1	1
1	0	0	0	0	1
2	0	0	0	1	0

and an arbitrary three-valued function $\phi(x_1, \ldots, x_n)$ become two-valued or inconsistent when any unprovable formula is added to the axioms. (N_2 was not primitive in the original but defined as $KNN\alpha C\alpha N\alpha$.) Thus the system has more possibilities of extension, by new cases of ϕ, than was originally envisaged, but fewer in terms of already axiomatized ϕ.

In the statement of the axioms i,j take values 1 or 2. The rules are detachment and substitution.

1. $CCCpqrCCrpCsp$
2_j. CpN_1N_jp
$3_{i,j}$. $CN_ipN_1N_jp \quad (i \neq j)$
4_i. CN_ipCpq
5_i. $CpCN_iqN_iCpq$
6. CCN_2ppCCN_1ppp
7. $Cx_1'Cx_2' \ldots . Cx_n'\phi(x_1, x_2, \ldots, x_n)' \quad (n \geq 0)$

7 prescribes the writing of 3^n axioms in correspondence with the 3^n lines of the truth-table of ϕ. In each, α' is α or $N_1\alpha$ or $N_2\alpha$ according as α has the value $0,1,2$ in the corresponding line of the table.

Received March 11, 1965