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Notre Dame Journal of Formal Logic
Volume VIII, Numbers 1 and 2, April 1967

A NOTE ON THE AXIOMATIZATIONS OF CERTAIN MODAL SYSTEMS[1]

ANJAN SHUKLA

In this note I prove

a) that the proper axiom of S2°, viz.,

B8 $\mathfrak{C}MKpqMp$

can be substituted by the following thesis

V1 $\mathfrak{C}MKpNpMp$

which shows that not only {S2°} ⇄ {S1°;*B8*} ⇄ {S1°;*V1*} but also, *a fortiori,* that {S2} ⇄ {S1;*B8*} ⇄ {S1;*V1*}; and
b) that a result of Hallden who has proved in [2], p. 128, Lemma 4, that the addition of

P1 $\mathfrak{C}MKpNpKpNp$

to S3 as a new axiom generates a system equivalent to S4 can be strengthened as follows:

The addition of a thesis

V2 $\mathfrak{C}MKpNpp$[2]

which, clearly, is an elementary consequence of *P1* as a new axiom to S3° gives a system equivalent to S4°, i.e., {S4°} ⇄ {S3°;*V2*} and, therefore, {S4} ⇄ {S3;*V2*}.

On the other hand I show that a result of Yonemitsu who has proved in [9] that {S2;*P1*} ⇄ {T} can also be strengthened; as follows: {S1;*V2*} ⇄ {T}. It may be remarked that this shows that *V2* is a weaker formula than the proper axiom of S4° or S4, i.e.,

C10 $\mathfrak{C}MMpMp$

This remark—that *V2* is weaker than C10—was made by Sobociński in [7].

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1. I am indebted to Professor Bolesław Sobociński for helpful suggestions.
2. The structural similarity of *V1* and *V2* may be noted.

Received July 23, 1965