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A DECISION PROCEDURE FOR FITCH'S PROPOSITIONAL CALCULUS

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In this paper[1] a *Sequenzenkalkül*, in the sense of Gentzen [3], will be formulated and shown equivalent (in a sense to be specified) to the propositional system (which we will term **F**) of Fitch's [2]. Naturally, the proof of equivalence requires an elimination theorem for the first system; the bulk of this paper, in fact, will concern itself with the task of establishing such a theorem. Finally, a decision method will be sketched for the *Sequenzenkalkül*, and thereby, indirectly, for Fitch's system. Though indirect and more complicated in some ways than the methods of James [4] and Resnik [7], this method has the advantage of applying to Fitch's full system of propositional calculus; the procedure of [4] does not take into account formulas containing nested implications, and that of [7] applies only to the implicational fragment of **F**.

1. *The System* **LF**. This is an **L**-system, in the sense of [3], designed to be equivalent to the system **F**.

1.1. *Wffs*. Any propositional variable p is well-formed (wf); furthermore, if A and B are wf, so are $(A \vee B)$, $(A \wedge B)$, $\sim A$, and $(A \supset B)$. Where α and β are strings of wffs separated by commas, $\alpha \vdash \beta$ is a (wf) *sequent*.

1.2. *Axioms*. There is one axiom-scheme, *identity* (**Id**): $A \vdash A$.

1.3. *Rules*.

1.3.1. *Structural rules*:

$$\vdash\mathbf{K}\ \frac{\alpha \vdash \beta}{\alpha \vdash A,\beta} \qquad \mathbf{K}\vdash\ \frac{\alpha \vdash \beta}{\alpha,A \vdash \beta}$$

$$\vdash\mathbf{C}\ \frac{\alpha \vdash \beta,A,B,\gamma}{\alpha \vdash \beta,B,A,\gamma} \qquad \mathbf{C}\vdash\ \frac{\alpha,A,B,\beta \vdash \gamma}{\alpha,B,A,\beta \vdash \gamma}$$

$$\vdash\mathbf{W}\ \frac{\alpha \vdash A,A,\beta}{\alpha \vdash A,\beta} \qquad \mathbf{W}\vdash\ \frac{\alpha,A,A \vdash \beta}{\alpha,A \vdash \beta}$$

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