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THE DEDUCTION THEOREM IN S4, S4.2, AND S5

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In a certain sense, there is no trick to merely stating the deduction theorem for a given system (on the assumption, of course, that it holds for that system). The general statement of the theorem might be, "*If there is a proof from the hypotheses* $A_1, \ldots, A_n$ *for the formula* B, *then there is a proof from the hypotheses* $A_1, \ldots, A_{(n-1)}$ *for the formula* $A_n \supset B$." The problem in formulating the deduction theorem lies not in simply stating it as above, but in defining just what we mean by "proof from hypotheses" for the system in question. Once we have such a definition, the statement and proof of the theorem will ordinarily present no real problem.

The three Lewis-modal systems with which we are concerned will be considered to be formulated on a **CPC** base, following, in general, Lemmon [2]. They will contain, first of all, any basis sufficient for the complete **CPC**, including the rules of substitution and detachment. Each of these systems will also contain the rule **RL**: "*If* α *is a theorem, so too is* $L\alpha$." The additional axioms are, for S4:

1. *CLCpqLCLpLq*
2. *CLpp*.

For S4.2, axioms 1. and 2. and also (see [3], p. 313):

3. *CMLpLMp*.

For S5, axioms 1. and 2. and also:

4. *CNLpLNLp*.

Since these systems are formulated on a **PC** base, we might suspect that a good part of the definition of "proof from hypotheses" for these systems will be exactly as for the **CPC**. This is the case; here we shall make use of Church's definition of "proof from hypotheses" for the **CPC** in [1], p. 87. The clauses of the definition as he states it are easily extended to our modal systems; we may thus present what will amount to most of our definitions:

A finite sequence of wffs $B_1, B_2, \ldots, B_m$ *is called a "proof from the hypotheses* $A_1, A_2, \ldots, A_n$*" if for each* i, $i \leqslant m$, *either*

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