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ALGEBRAIC INDEPENDENCE IN AN INFINITE STEINER TRIPLE SYSTEM

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In a recent note [1] W. J. Frascella has given an effective construction of a Steiner triple system on a set of the power of the continuum. With every Steiner triple system an idempotent binary operation is associated in a natural way. The triple system can be regarded as an algebra, and one can consider the algebraic independence in the sense of Marczewski [2] on it.

Frascella's triple system gives rise to an algebra in which every two elements are independent, while every three of them are dependent. Thus the numerical characteristics ι and ι^* introduced by Marczewski in [3] for finite algebras are both equal 2 here. The purpose of the present paper is to prove these facts.

1. A Steiner triple system on a set S is a class of three-element sets (called *Steiner triples*) such that every pair of elements of S belongs to exactly one Steiner triple. A given Steiner triple system on S determines a binary operation on S such that

(1) $$x \circ x = x ,$$

and for $x \neq y$, $x \circ y$ is the third element of the triple determined by x and y. Hence the binary operation "$\circ$" has the following additional properties

(2) $x \circ y = y \circ x$,
(3) $x \circ (x \circ y) = y$, $\quad y \circ (x \circ y) = x$,
(4) if $x \neq y$ then $x \neq x \circ y \neq y$.

2. Consider the algebra $<S,\circ>$ with the single fundamental operation "$\circ$".

Proposition. *Every algebraic binary operation* $\mathsf{f}(x,y)$ *of the algebra* $<S,\circ>$ *is either* $x \circ y$ *or one of the trivial operations* $\mathsf{e}_1^2(x,y) = x$, $\mathsf{e}_2^2(x,y) = y$.

Proof. The binary operation $\mathsf{f}(x,y)$ can be expressed as a word consisting of the letters x, y, the symbol $\circ$ and brackets. The number of letters in the word is called its length. An operation which can be

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