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COMBINATORIAL DESIGNS ON INFINITE SETS

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I. INTRODUCTION

§1. *Generalities*[1]. Roughly speaking, the field of combinatorial mathematics can be said to deal with those problems of arranging objects according to some fixed pattern and in determining how many distinct ways this can be accomplished[2]. Observe that no restriction is placed on the set to which the objects can be considered to belong. Consequently, a combinatorics of the infinite naturally evolves when investigations are concerned with arrangements of sets which are not finite. Frequently it happens that a meaningful question of a combinatorial nature, initially posed with reference to a finite collection, retains its interest when one allows the collection to be infinite. The generalization is usually realized by permitting one or more of those symbols, which represents a natural number in the finite formulation of the problem, to now stand for an arbitrary cardinal number. Very often, however, such a simplistic generalization trivializes a very interesting finite problem. In some cases, therefore, to recapture the spirit of a finite combinatorial problem in the infinite case, it is necessary to effect more sophisticated alterations in the hypotheses of the original problem.

In the course of the present report this method of generalization will be exhibited. Our interests will converge upon a single, yet important, area of combinatorial research: the existence and construction of designs. Design problems in combinatorics, which are both intriguing and difficult, almost always deal with arrangements of finite sets. The aim, herewith, is to develop a theory of combinatorial designs on infinite sets which bears a

1. The present researches constitute a part of the author's doctoral dissertation, *Block Designs On Infinite Sets*, written under the direction of Professor B. Sobociński and accepted by the University of Notre Dame in partial fulfullment of the requirements of the degree of Ph.D. in Mathematics, February, 1966.
2. M. Hall [5] asserts this as a working definition of combinatorial analysis.

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