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RINGS OF TERM-RELATION NUMBERS AS NON-STANDARD MODELS

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1. *Purpose*. The concept of ultraproduct introduced by Łoś [3] has proved to be an important tool in model theory, as shown by A. Robinson's non-standard analysis, for example [4]. But in spite of the ultraproduct's usefulness, Vaught has pointed out its dramatic limitations and the essential need for other ways of developing models [5, p. 311]. With these points in mind, we shall show how systems of term-relation numbers can be used as model-theoretic devices. We hope this will have the heuristic value of suggesting other ways to construct finitary models that will contrast with the idea of infinite direct product basic to Łoś's theory. The systems mentioned in this paper have the following algebraic limitation (which may not be decisive): they are commutative rings with zero divisors, and since the least ideal that contains all those zero divisors is the whole ring, it is not possible to map these rings into a field through the usual method of forming the difference ring. This leaves unanswered the question of whether or not rings of term-relation numbers can be mapped nontrivially into an integral domain by some other method. Since we know so much more about integral domains than we do about ordered rings with zero divisors, the usefulness of the systems described here as generators of non-standard models of analysis remains undetermined. Nevertheless, these systems are non-standard models of certain areas of arithmetic and analysis.

2. *Rings of real term-relation numbers, order, finite infinitesimals*. A reading of [1] and [2] is necessary to follow the forthcoming discussion. In [2] the ring $T^{\infty'}$ is introduced; it is composed of term-relation numbers whose final components are drawn from the whole integral domain of ordinary integers. The paper also describes a generalization to rings of rational term-relation numbers. The corresponding definitions may now be broadened to introduce real and complex term-relation numbers. Details are left to the reader. The systems obtained are similar to those defined in [2], i.e., they are all commutative rings without identity and with proper zero divisors. Although most of the following considerations apply to both

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