Notre Dame Journal of Formal Logic
Volume VIII, Numbers 1 and 2, April 1967

# LIMITED UNIVERSAL AND EXISTENTIAL QUANTIFIERS IN COMMUTATIVE PARTIALLY ORDERED RECURSIVE ARITHMETICS

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1. In this paper we shall be dealing with the two different types of recursive arithmetics which will be described as V-systems and C-systems. These arithmetics have the following properties.

V-*systems*

(1) Every number $x$ has $n$ successors, denoted by $S_1x$, $S_2x$, . . . , $S_nx$.
(2) The system has three initial functions, namely, the zero function, $Z(x)$, written $0$, the identity function, $I(x)$, written $x$, and n successor functions, $S_vx$, with $v = 1, 2, \ldots, n$.
(3) Primitive recursive functions can be defined by using the schema

$$F(x, 0) = a(x)$$
$$F(x, S_vy) = b_v(x, y, F(x, y)) \quad v = 1, 2, \ldots, n,$$

where $a(x)$ and $b_v(x,y)$ are previously defined functions. Functions can also be defined explicitly by substitution.
(4) The system is made commutative by introducing the axiom

$$S_vS_ux = S_vS_ux \qquad u, v = 1, 2, \ldots, n,$$

and by stipulating that the functions used in a defining schema of the type given above satisfy the condition

$$b_v(x, S_uy, b_u(x, y, F(x, y))) = b_u(x, S_vy, b_v(x, y, F(x, y))).$$

C-*systems*

(1) The elements of the system are ordered sets of $n$ natural numbers, written $(x_1, x_2, \ldots, x_n)$.
(2) Functions are defined as ordered sets of $n$ primitive recursive functions in single successor recursive arithmetic, written

$$(f_1(x_1, \ldots, x_n), f_2(x_1, \ldots, x_n), \ldots, f_n(x_1, \ldots, x_n)).$$

The functions $f_1, f_2, \ldots, f_n$ are called component functions.
(3) Two functions in a C-system are said to be equal if their corresponding component functions are equal, i.e.

*Received February 4, 1966*