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ON THE INDEPENDENCE OF CERTAIN AXIOMS IN THE DEFINITION OF AN m-ARRANGEMENT

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It was shown in [1] that of the axioms defining an m-arrangement, i.e. **3.1-3.9, 3.4, 3.5** and **3.9** are independent. The purpose of this note is to show that after certain trivial modifications, **3.1, 3.2** and **3.3** are also independent. **3.7** and **3.8** are known to be independent as a pair [1]. Nothing is yet known about the independence of **3.6**; indeed, this particular problem seems to be extremely complex.

The following modifications of **3.1-3.9** are called for. They in no way affect any of the proof in [1].

3.5: *If* x,y *and* z *are distinct points of a 1-flat and if* $\overline{xy}$, $\overline{xz}$ *and* $\overline{yz}$ *all exist, then* $\overline{xy} \cup \overline{yz} = \overline{xy}, \overline{yz}$, *or* $\overline{xz}$.

3.6: *If* $S = \{x_0, \ldots, x_k\}$ *is linearly independent and* $k \geqslant 2$, *then . . .*

3.7: *If* $C(S)$ *is a* k*-simplex*, $k \geqslant 2$, *. . .*

3.8: *If* $C(S)$ *is a* k*-simplex*, $k \geqslant 2$, *. . .*

If **3.1-3.5** hold, then **3.6-3.8** are theorems for $k = 1$.

The independence of **3.1**: Let $X = [0,1]$, the closed interval between 0 and 1 with its usual topology. Set $F^0 = \{\{x\} | x \epsilon (0,1)\} \cup \{\{0,1\}\}$, $G = \{F^{-1}, F^0\}$.

The independence of **3.2**: Since the definition of a topological geometry involves two distinct assumptions, two counterexamples are required:

a) Each flat is closed: Let $X = \{a,b,c\}$ with the topology $\{\{a,b,c\}, \{a,b\}, \{b,c\}, \{b\}, \phi\}$. Set $F^0 = \{\{x\} | x \epsilon X\}$, $G = \{F^{-1}, F^0\}$.

b) The intersection of any family of convex sets is convex: Let $X = \{(x,y) | x^2 + y^2 = 1\} \subset R^2$ with the usual topology. Set $F^0 = \{\{x\} | x \epsilon X\}$, $G = \{F^{-1}, F^0\}$.

The independence of **3.3**: Let X be the set of real numbers with the discrete topology. Set $F^0 = \{\{x\} | x \epsilon X\}$, $G = \{F^{-1}, F^0\}$.

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