Notre Dame Journal of Formal Logic
Volume VIII, Number 3, July 1967

A CHARACTERIZATION OF $\mathbf{S}^m$ BY MEANS OF TOPOLOGICAL GEOMETRIES

MICHAEL C. GEMIGNANI

In a recent paper in this Journal [1], the author characterized $\mathbf{R}^m$ as a topological space using the concept of a topological geometry. The purpose of the present paper is to present a similar characterization for the m-sphere $\mathbf{S}^m$. The terminology and propositions referred to by number are those of [1].

Theorem 1: Let X and G form an m-arrangement, $m \geq 2$, and suppose X is second countable. Then if $S=\{x_0, \ldots, x_m\}$ is a linearly independent subset of X and $T=\{p_0, \ldots, p_m\}$ is any maximal linearly independent subset of $\mathbf{R}^m$ with the usual Euclidean geometry $\overline{G}$, then there is a homeomorphism d which maps $C(S)$ onto $C(T)$ and $F^iC(S)$ onto $F^iC(T)$, $i=0, \ldots, m$, such that $d(G_{C(S)}) = \overline{G}_{C(T)}$.

Proof: Set $d(x_i)=p_i$, $i = 0, \ldots, m$. Let $S_1 = \bigcup_{i<j} \overline{x_i x_j}$. By **3.27**, $d|S$ can be extended to $d_1: S_1 \rightarrow K^1C(T)$, the *1*-skeleton of $C(T)$ such that d_1 is a homeomorphism onto which carries $\overline{x_i x_j}$ onto $\overline{p_i p_j}$. Set $S_2 = \bigcup_{i<j<k} C(\{x_i, x_j, x_k\})$. Define $d_2: S_2 \rightarrow K^2C(T)$, the *2*-skeleton of $C(T)$ as follows: If $C(\{x_i, x_j, x_k\}) \subseteq S_2$, $d_2 = d_1$ on $\mathrm{Bd}\,C(\{x_i, x_j, x_k\})$. Choose $z \in \mathrm{Int}\, C(\{x_i, x_j, x_k\})$. Then $\mathbf{f}_1(x_i, z)$

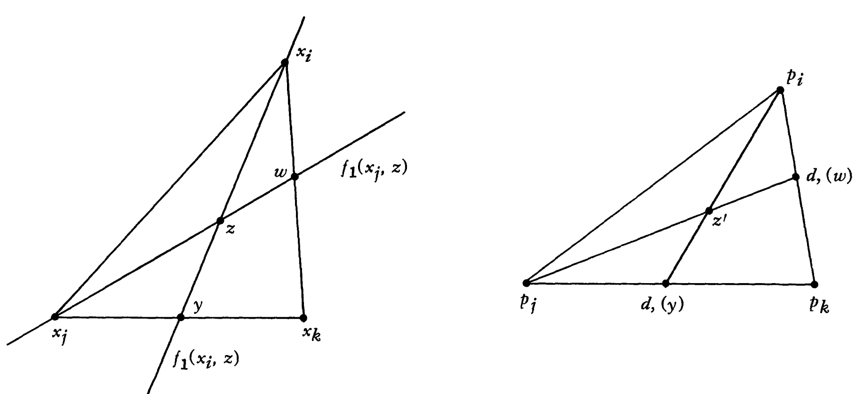

Fig. 1.

Received July 7, 1965