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A HENKIN COMPLETENESS THEOREM FOR T

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In [1] A. Bayart uses a method similar to that of Henkin [2] to prove a completeness theorem for the S5 modal predicate calculus.[1] We show how this method can be adapted to give completeness results for first order quantificational T and S4 with the *Barcan* formula.[2] T is a modal predicate calculus with propositional variables p, q, r . . . etc., individual variables x, y, z . . . etc., individual constants u_1, u_2, u_3 . . . etc., and predicate variables ϕ, ψ, χ etc., $\sim$, $\vee$, the universal quantifier and L (the necessity symbol). We assume usual formation rules and definitions of $\supset$, . , $\equiv$, $\exists$, and M. T has the following axioms and axiom schemata,

PC some set sufficient for the propositional calculus

LA1 $Lp \supset p$
LA2 $L(p \supset q) \supset (Lp \supset Lq)$

$\forall_1$ $(a)\alpha \supset \beta$ where a is an individual variable and β differs from α only in having some individual symbol b (variable or constant) everywhere where a occurs free in α provided a in α does not occur within the scope of (b).
B (the *Barcan* formula) $(x)L\alpha \supset L(x)\alpha$ where α is any wff. and the following rules of transformation; Uniform substitution for propositional variables provided no variable is bound as a result of substitution. (If PC and LA1, LA2 are formulated as schemata this rule, and the propositional variables, are unnecessary)

MP $\vdash \alpha, \vdash \alpha \supset \beta \rightarrow \vdash \beta$
LR1 (Necessitation) $\vdash \alpha \rightarrow \vdash L\alpha$
$\forall_2$ $\vdash \alpha \supset \beta \rightarrow \vdash \alpha \supset (a)\beta$ where a is some variable not free in α.

We obtain S4 by adding LA3 $Lp \supset LLp$ and S5 by adding LA4 $\sim Lp \supset L \sim Lp$ (If we have LA4 we may drop the *Barcan* formula; cf. [6]).

We say that a formula is *closed* (a cwff) if it contains no free variable. Where Λ is a set of formulae and β a wff we say that $\Lambda \vdash \beta$ iff there is some finite subset of Λ, $\{\alpha_1, \ldots, \alpha_n\}$ such that $(\alpha_1 \ldots \alpha_n) \supset \beta$. The following are derivable;

T1 (The Deduction Theorem) *If* Λ, $\alpha \vdash \beta$ *then* $\Lambda \vdash (\alpha \supset \beta)$.

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