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PROBABILITY AS DEGREE OF POSSIBILITY

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Let L be a set of sentences closed under disjunction and negation. Let S be a system of possible worlds. For p in L and W in S, let "$W(p) = \mathbf{T}$" mean that p is true in W. We assume that

(1) If $W(p \vee q) = \mathbf{T}$, then $W(p) = \mathbf{T}$ or $W(q) = \mathbf{T}$.
(2) If $W(p) = \mathbf{T}$, then $W(-p) \neq \mathbf{T}$.
(3) If p is truth functionally (**tf**) valid (and hence necessary), then $W(p) = \mathbf{T}$ for every W in S.

(S could be identified with a system of Hintikka model sets of sentences of L.)

Let N be the number of worlds in S, and assume for now that N is *finite* and positive. For any p in L, let $\mathsf{N}(p)$ be the number of worlds W in S such that $W(p) = \mathbf{T}$. Define the probability that p by

$$\pi(p) = \mathsf{N}(p)/\mathsf{N}.$$

Then for any p in L

(4) $0 \leq \pi(p) \leq 1$
(5) $\Diamond p$ *iff* $\pi(p) > 0$
(6) $\Box p$ *iff* $\pi(p) = 1$.

Now

(7) *If* $W(p) = \mathbf{T}$, *then* $W(p \vee q) = \mathbf{T}$,

for $-p \vee (p \vee q)$ is **tf** valid, so by (3), $W(-p \vee (p \vee q)) = \mathbf{T}$, and then by (1), $W(-p) = \mathbf{T}$ or $W(p \vee q) = \mathbf{T}$. But $W(p) = \mathbf{T}$, so by (2), $W(-p) \neq \mathbf{T}$, and thus $W(p \vee q) = \mathbf{T}$. Next:

(8) *If* $-\Diamond(p \wedge q)$, *then* $\pi(p \vee q) = \pi(p) + \pi(q)$.

If $W(p \vee q) = \mathbf{T}$, then by (1) and the impossibility of $(p \wedge q)$, exactly one of p, q is true in W. By (7), if $W(p) = \mathbf{T}$ or $W(q) = \mathbf{T}$, then $W(p \vee q) = \mathbf{T}$. Hence $\{W \epsilon S \mid W(p \vee q) = \mathbf{T}\}$ is the disjoint union of $\{W \epsilon S \mid W(p) = \mathbf{T}\}$ and $\{W \epsilon S \mid W(q) =$

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