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A SEMI-LATTICE THEORETICAL CHARACTERIZATION OF ASSOCIATIVE NEWMAN ALGEBRAS

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The aim of this note[1] is to stress a fact which, due to the original formulation of Newman's systems given in [1], can be easily overlooked: an associative Newman algebra can be considered as a semi-lattice with respect to the binary operation $\times$ to which the additional postulates are added concerning the properties of the binary operation + (which is neither a lattice-theoretical join nor a lattice theoretical symmetrical difference) and the unary operation -, i.e., the complementation peculiar to this system. Namely, it will be shown that in the field of the axioms *A1*-*A11* the proper axioms of system $\mathfrak{D}$ of associative Newman algebra, *cf.* [2], section **3**, i.e., the postulates

F1 $[ab]:a,b\in B\,.\supset.\,a = a + (b\times\overline{b})$
F2 $[ab]:a,b\in B\,.\supset.\,a = a\times(b+\overline{b})$
H1 $[abc]:a,b,c\in B\,.\supset.\,a\times(b+c) = (c\times a)+(b\times a)$
L1 $[abc]:a,b,c\in b\,.\supset.\,a\times(b\times c) = (a\times b)\times c$

are inferentially equivalent to the following formulas: *F1*, *F2*, *L1* and

F33 $[ab]:a,b\in B\,.\supset.\,a\times b = b\times a$
C1 $[abc]:a,b,c\in B\,.\supset.\,a\times(b+c) = (a\times b)+(a\times c)$

and, moreover, that the idempotent law with respect to operation $\times$, i.e.,

F7 $[a]:a\in B\,.\supset.\,a = (a\times a)$

is a consequence of the axioms *F1*, *F2* and *C1*.

Proof: In [2], section **3**, it has been proved that the formulas *F33* and *C1* follow from *F1*, *F2*, *H1* and *L1*. On the other hand, let us assume *F1*, *F2*, *L1*, *F33* and *C1*. Then:

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1. An acquaintance with the papers [1], [2] and [3] is presupposed. An enumeration of the formulas discussed in this note is the same which they have in [3] and [2]. The axioms *A1*-*A11*, *cf.* [3], section 1, will be used tacitly in the deductions presented in this note.

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