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AN APPLICATION OF MATHEMATICAL LOGIC TO THE INTEGER LINEAR PROGRAMMING PROBLEM

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The general integer linear programming problem, namely optimising a linear function subject to non-negative integer solutions of a set of simultaneous linear inequalities, was until 1958 one of the major unsolved problems in the theory of linear programming. Yet, as I shall show in this paper, a technique for solving this problem was available in 1929 in the paper by M. Presburger [1] which provided a decision procedure for a certain fragment of recursive arithmetic. Of course it must be said that the subject of linear programming did not exist in 1929 and so Presburger's algorithm was in this context a solution to a non-existent problem. I shall describe briefly Presburger's result and then show that the existence of an algorithm for solving the integer linear programming problem is an obvious consequence of it. I make no claims for the algorithm as a practical means of computation when compared with the solution due to R. E. Gomory in 1958 [2], or any subsequent one. It is however a simple method of showing that the problem is soluble and predates Gomory's solution by almost 30 years.

Presburger provided a decision procedure (i.e. a procedure for deciding whether statements were true or false) for the formal system of arithmetic referred to as system D in Hilbert and Bernays Vol. I [3]. System D refers to the 1st order theory with the one predicate, equality, one constant, 0, and two functions, S (the successor function) and the addition function. By applying the function S to the constant 0 we have all the natural numbers in the system, and applying the addition function to the variables and constants we obtain as the terms of the system, linear forms in any number of variables with positive coefficients and constants. The atomic formulae of the system are the expressions $s = t$ where s, t are terms, and hence the atomic formulae are just linear equations with the variables on their positive side.

Thus the system contains the propositional connectives for "not"

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