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MATTERS OF SEPARATION

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1. Extending in some respects, sharpening in others, results in the literature, we establish here that:

(1) *Every classically valid wff A of* QC=, *the first-order quantificational calculus with identity, is provable by means of axiom schemata* A1-A3 *and rule* R1 *in Table I, plus the axiom schemata and rules of that table for only such of the logical symbols* '∼', '&', 'v', '≡', '∀', '∃', *and* '=' *as occur in A*,

(2) *Every intuitionistically valid wff A of* QC= *is provable by means of axiom schemata* A1-A2 *and rule* R1 *in Table I, plus the axiom schemata and the seven rules of that table for only such of the logical symbols in question as occur in A*.

In the first of our two theorems R2 is to serve as rule for '∀'; in the second, R2 or R2′ according as '&' occurs or not in *A*.

TABLE I

Axiom schemata

For '⊃':	A1.	$A \supset (B \supset A)$
	A2.	$(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
	A3.	$((A \supset B) \supset A) \supset A$
For '∼':	A4.	$(A \supset B) \supset (\sim B \supset \sim A)$
	A5.	$A \supset \sim\sim A$
	A6.	$\sim\sim A \supset (\sim A \supset B)$
For '&':	A7.	$(A \,\&\, B) \supset A$
	A8.	$(A \,\&\, B) \supset B$
	A9.	$A \supset (B \supset (A \,\&\, B))$
For 'v':	A10.	$A \supset (A \vee B)$
	A11.	$B \supset (A \vee B)$
	A12.	$(A \supset C) \supset ((B \supset C) \supset ((A \vee B) \supset C))$
For '≡':	A13.	$A \supset ((A \equiv B) \supset B)$
	A14.	$A \supset ((B \equiv A) \supset B)$
	A15.	$(A \supset B) \supset ((B \supset A) \supset (A \equiv B))$
For '∀':	A16.	$(\forall X)\, A \supset A(Y/X)$
For '∃':	A17.	$A(Y/X) \supset (\exists X)\, A$

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