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PREMISSES ARE NOT AXIOMS

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On page 35 of his excellent *Mathematical Logic* [1], Stephen C. Kleene writes: "For the propositional calculus applied to infer formulas from assumptions $A_1, \ldots, A_m$, the formulas $A_1, \ldots, A_m$ are in effect allowed to function as axioms also." Kleene does not go on to call assumption formulas axioms. Still his suggestion that assumption formulas function as axioms is misleading. And I think that Kleene's way of putting what we do when we use assumption formulas is fairly common amongst logic instructors. Hence, this note.

For instance, Kleene's suggestion could lead someone to the following misconception about what they do when they establish that $(P \supset Q)$, $(Q \supset R) \therefore (P \supset R)$ is a derived rule in some axiomatization of classical propositional calculus. Let us say that they—those with the misconception—have axiom schemata which together with *Modus Ponens* suffices for a deductively complete consistent axiomatization of classical propositional logic. Call the set of axiom schemata AS. They may think that when they establish that a formula schema $(P \supset R)$ can be derived from formula schemata $(P \supset Q)$ and $(Q \supset R)$, they add $(P \supset Q)$ and $(Q \supset R)$ as axiom schemata to AS to get a larger set of axiom schemata AS' and have $(P \supset R)$ as a theorem schema form AS'. Now, of course, this has to be a totally erroneous conception of what we do when we establish that $(P \supset Q)$, $(Q \supset R) \therefore (P \supset R)$ is a derived rule in an axiomatization of classical propositional calculus. It has to be erroneous because a derivation of $(P \supset R)$ from $(P \supset Q)$ and $(Q \supset R)$ does not require an inconsistent system. But it is well known that if a non-tautologous formula schema is added as an axiom schema to a complete, consistent classical propositional calculus the resulting system in inconsistent. (For a proof of this see Kleene [2], page 134.)

When we use assumption formulas they do not function as axioms in the sense that we first add assumption formulas to our axiom schemata and then proceed to construct demonstrations. We do not add assumption formulas to the axiom schemata. We first place the assumption formulas directly into demonstrations and then add axioms and use rules of proof to

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