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CERTAIN SETS OF POSTULATES FOR DISTRIBUTIVE LATTICES WITH THE CONSTANT ELEMENTS

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The single aim of this note is to establish such axiomatizations of distributive lattice with the constant elements, i.e. either with I and O, or with I only or with O only, that each of the equational axiom-systems presented here will contain one and only one axiom in which no constant element occurs. Since the constructions of such axiomatizations are related to certain results previously obtained and published by some other authors, the involved investigations will be referred to briefly in section **1**.

1 G. D. Birkhoff and G. Birkhoff have established, *cf.* [1], [2], pp. 135-137, and [3], pp. 34-35, that *any algebraic system*

$$\mathfrak{D} = \langle A, \cap, \cup, I\rangle$$

with two binary operations $\cap$ *and* $\cup$, *and with one constant element* $I \epsilon A$ *which satisfies the following seven postulates*

K1 $[a] : a \epsilon A \,.\supset .\, I = a \cup I$
K2 $[a] : a \epsilon A \,.\supset .\, I = I \cup a$
K3 $[a] : a \epsilon A \,.\supset .\, a = a \cap I$
K4 $[a] : a \epsilon A \,.\supset .\, a = I \cap a$
K5 $[a] : a \epsilon A \,.\supset .\, a = a \cap a$
K6 $[abc] : a, b, c \epsilon A \,.\supset .\, a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$
K7 $[abc] : a, b, c \epsilon A \,.\supset .\, (b \cup c) \cap a = (b \cap a) \cup (c \cap a)$

is a distributive lattice with I.

In [4], pp. 26-27, Croisot has shown that these axioms are mutually independent, *cf.* [2], p. 139, problem 65, and, moreover, he has proved that the axioms *K1*-*K7* are inferentially equivalent to the axioms *K1*, *K3*, *K5* and

L1 $[abc] : a, b, c \epsilon A \,.\supset .\, a \cap (b \cup c) = (c \cap a) \cup (b \cap a)$

2 Theorem 1. *Any algebraic system*

$$\mathfrak{A} = \langle A, \cap, \cup, I, O\rangle$$

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