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# A NOTE ON IMMUNE SETS

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In [1] the following sequence of theorems were proved, which were clearly inspired by the fact that an infinite set is immune iff it has no infinite recursively enumerable subset.* (Throughout this paper "set" means "subset of $\mathfrak{N}$" where $\mathfrak{N}$ denotes the set of non-negative integers.)

Theorem 1: *Let A be an infinite set. Then*:

(a) *A is hyperimmune iff it has no strongly finitely almost recursively enumerable infinite subset*
(b) *A is hyperhyperimmune iff it has no finitely almost recursively enumerable infinite subset*
(c) *A is strongly hyperhyperimmune iff it has no almost recursively enumerable infinite subset.*

Definitions of these concepts may all be found in [1]; however we recall the most important of them here. An injective total function on $\mathfrak{N}$ is called *almost recursive* if its inverse has a partial recursive extension. A set is called almost recursively enumerable (hereafter abbreviated a.r.e.) if it is finite or the range of an almost recursive function; and is called almost recursive (abbreviated a.r.) if it is finite or the range of a strictly increasing almost recursive function. If $f(x)$ is a recursive function, the sequence of sets

$$\omega_{f(0)}, \omega_{f(1)}, \ldots$$

is called a disjoint array if $\bigwedge_i \bigwedge_j [i \neq j \rightarrow \omega_{f(i)} \cap \omega_{f(j)} = \phi]$ and $\bigwedge_i [\omega_{f(i)} \neq \phi]$. It is called a finite disjoint array if in addition: $\bigwedge_i [\omega_{f(i)}$ is a finite set].

We shall use an effective enumeration of all finite sets denoted by

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