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NOTE ON INDUCTIVE FINITENESS IN MEREOLOGY

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In this note we prove that for inductive finiteness,

$$[a]: \mathsf{Fin}\{a\} . \supset . \ \mathsf{Fin}\{\mathbf{st}(a)\} .$$

Sobociński proved this previously under the added hypothesis, $\mathsf{dscr}\{a\}$. Theorems quoted, but not stated in this note, refer to the Mereological Preliminaries in [1]. We shall also need the following well-known definitions and properties concerning inductive finiteness.

DF1. $[a\varphi] .\because . \varphi\{\wedge\} : [Ab] : A \,\varepsilon\, a \,.\, \varphi\{b\} . \supset . \ \varphi\{b \cup A\} :\equiv . \mathsf{InR}\langle\varphi\rangle \{a\}$
DF2. $[a] .\because . [\varphi] : \mathsf{InR}\langle\varphi\rangle \{a\} . \supset . \ \varphi\{a\} :\equiv . \ \mathsf{Fin}\{a\} .$
F1. $[A] : \neg\{A\} . \supset . \ \mathsf{Fin}\{A\} .$
F2. $[ab] : \mathsf{Fin}\{a\} . \ \mathsf{Fin}\{b\} . \supset . \ \mathsf{Fin}\{a \cup b\} .$
F3. $[ab] : a \overset{\infty}{\underset{\infty}{\ }} b \,.\, \mathsf{Fin}\{b\} . \supset . \ \mathsf{Fin}\{a\} .$

We begin the proof with some auxiliary definitions.

D1. $[ACb] : C \,\varepsilon\, \varphi\,(bA) . \equiv . \ C \,\varepsilon\, b \,.\, C \,\varepsilon\, \mathbf{el}(A) .$
T1. $[bA] . \varphi(bA) \subset b .$ [*D1*]
D2. $[ABCb] : C \,\varepsilon\, \Phi\langle bB\rangle\,(A) . \equiv . \ C \,\varepsilon\, \mathbf{Kl}(\varphi(bA)) . A \,\varepsilon\, \mathbf{Kl}(\varphi(bA) \cup B) .$
$A \,\varepsilon\, \mathbf{st}(b \cup B) \backslash (\mathbf{st}(b) \cup B) .$
D3. $[BCb] : C \,\varepsilon\, \Phi\,(bB) . \equiv . \ C \,\varepsilon\, C \,.\, [\exists A] . \ C \varepsilon\, \Phi\langle bB\rangle\,(A) .$
T2. $[BCb] : C \,\varepsilon\, \Phi\,(bB) . \equiv . \ [\exists A] . \ C \,\varepsilon\, \Phi\langle bB\rangle\,(A) .$ [*D3*]
T3. $[Bb] . \Phi\,(bB) \subset \mathbf{st}(b) .$ [*D3*; *D2*; *T1*; *M18*]
T4 $[BCb] : C \,\varepsilon\, \Phi\,(bB) . \supset . [\exists A] . \ C \,\varepsilon\, \Phi\langle bB\rangle\,(A) . A \,\varepsilon\, \mathbf{st}(b \cup B) \backslash (\mathbf{st}(b) \cup B) .$
[*D3*; *D2*]
T5. $[AB] : \neg\{B\} . \ A \,\varepsilon\, \mathbf{Kl}(B) . \supset . A \,\varepsilon\, B .$
$[AB] : \mathrm{Hyp}\,(2) . \supset .$
$[\exists C] .$
3) $C \,\varepsilon\, B .$ [*M10*; 2]
4) $B \,\varepsilon\, C .$ [3; 1]
5) $B = \mathbf{Kl}(B) .$ [*M12*; 4]
$A \,\varepsilon\, B .$ [5, 2]
T6. $[ABb] : \neg\{B\} . A \,\varepsilon\, \mathbf{st}(b \cup B) \backslash (\mathbf{st}(b) \cup B) . \supset . [\exists C] . \ C \,\varepsilon\, \Phi\langle bB\rangle\,(A) .$
$C \,\varepsilon\, \Phi\,(bB) .$

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