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A PROOF OF A THEOREM OF ŁUKASIEWICZ

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We present a proof of Łukasiewicz's theorem in [1] that Syllogism, Peirce and any expression of the form $CpC\alpha\beta$ are, with detachment and substitution, sufficient for two-valued implication. Łukasiewicz proceeded by deriving $CpCqp$ with masterly economy of detachments, to get the Bernays axioms. The present method obtains instead $CpCCqqCqq$ to get the Wajsberg axioms from [2]. Though greatly more prodigal of detachments, it seems pleasing in virtue of a certain simplicity in the use of auxiliary theses derived only from 1 and 2, and by its delay of the use of 3 till the last moment. It may perhaps be found easier to remember than Łukasiewicz's and to be useful for purposes of instruction. The axioms are:

1. $CCpqCCqrCpr$
2. $CCCpqpp$
3. $CpC\alpha\beta$

From 1 and 2 we derive:

$D1.1 = 4.$	$CCCCqrCprsCCpqs$
$D4.4 = 5.$	$CCpCqrCCsqCpCsr$
$D4D5.1 = 6.$	$CCpqCCspCCqrCsr$
$D4.2 = 7.$	$CCpCpqCpq$
$D1.2 = 8.$	$CCprCCCpqpr$
$D6.2 = 9.$	$CCsCCpqpCCprCsr$
$D1D4.8 > 10.$	$CCCRRsCCCRQRs$
$D1DD1.6.8 > 11.$	$CCCCRQRsCQs$

10 and 11 are not the most general results of the detachments but substitutions in those. Q abbreviates Cqq, R abbreviates CQQ. 4 and 8 will not be used again, 6 only once.

In the following list, each member implies its successor in accordance with the thesis mentioned, the first therefore implies the last by successive applications of 1.

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