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A GENERALIZATION OF THE GENTZEN HAUPTSATZ

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1. Gentzen rules describe proofs in first order logic on the basis of simple transformations, each one related to the meaning of some logical operation, which are enough to avoid the use of the cut rule. A fundamental application of the rules is the reduction of predicate logic to propositional logic via the so-called extended Gentzen Hauptsatz or midsequent theorem (also called the Herbrand-Gentzen theorem in [1]). We present in this paper another reduction which seems very convenient for consistency proofs of universal theories. It is shown that if such a theory is inconsistent then some inconsistency can be found just using the cut rule to eliminate atomic formulas.

In the system considered by Gentzen only one kind of axiom is admitted: those of the form $A \rightarrow A$. We allow other axioms and prove the elimination of the cut rule under certain restrictions. The resulting system provides a good frame for the description of axiomatic theories and preserves the characteristic symmetry of the Gentzen rules.

2. We shall deal with some given first order language in which we assume the following symbols: free individual variables, bound individual variables, individual constants, function letters, the equality symbol, predicate letters, propositional connectives: $-$, $\vee$, $\wedge$, $\supset$, $\equiv$, and quantifiers: $\forall$, $\exists$.

Letters $a, b, c, \ldots$ are used as syntactic variables for free individual variables and letters $x, y, z, \ldots$ for bound individual variables. Terms are defined in the usual way. Atomic formulas are either expressions of the form $R(t_1, \ldots, t_n)$ where R is some predicate letter and $t_1, \ldots, t_n$ are terms, or expressions of the form $t = h$ where t and h are terms. Formulas are defined by induction in the usual way.

Letters $M, N, P, \ldots$ will denote finite (possible empty) sequences of formulas. The formulas of the sequence M are called components of M. We introduce a symbol $\rightarrow$ and expressions of the form $M \rightarrow N$ are called sequents.

The notation $A(t/b)$ denotes the result obtained when the term t is substituted for the variable b in the formula A and it is also a formula. If

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