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THE CONSISTENCY OF THE AXIOMS OF ABSTRACTION AND EXTENSIONALITY IN A THREE-VALUED LOGIC

ROSS T. BRADY

The Abstraction Axiom I want to consider is the following one, which is based on the Łukasiewicz three-valued logic.

$$(*)\ (Sy)(Ax)(x\,\varepsilon\, y \leftrightarrow \phi(x, z_1, \ldots, z_n))$$

where ϕ is either a propositional constant or constructed from atomic wffs $u\,\varepsilon\, v$ by using $\sim$, &, A. The connectives and quantifiers of the logic can be represented as follows:

	$p \,\&\, q$			$\sim p$	$p \vee q$			$p \rightarrow q$			$p \leftrightarrow q$			$p \supset q$		
p/q	1	$\frac{1}{2}$	0		1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0
1	1	$\frac{1}{2}$	0	0	1	1	1	1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	1	1
0	0	0	0	1	1	$\frac{1}{2}$	0	1	1	1	0	$\frac{1}{2}$	0	1	1	1

$(Ax)\,fx$ has the minimum value of the values of fx. $(Sx)\,fx$ has the maximum value of the values of fx.

Th. Skolem has produced models, in [1] and in [2] for an Abstraction Axiom the same as (*) except that ϕ may not be constructed using quantifiers A and S. He shows that the Axiom of Extensionality is also valid in his model in [2]. The procedure we use for constructing the model roughly follows the lines of P. C. Gilmore's paper (see [3]), where he constructed a model for his partial set theory PST′.

1. To construct the model, we need to extend the wffs used above to express (*) by adding some terms, some of which will be used as the domain of the model. We give the formation rules for terms and wffs as follows:

1. If x and y are set variables, then $x\,\varepsilon\, y$ is an atomic wff.
2. Any combination of wffs using $\sim$, $\rightarrow$, A are wffs.
3. A propositional constant (i.e., 1, $\frac{1}{2}$ or 0) is an atomic wff.

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