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SOME RESULTS ON GENERALIZED TRUTH-TABLES

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A generalized truth-table (also called a model) is an algebraic structure $\mathfrak{M} = (A, D, \Omega)$ such that A is a non-empty set of elements, D is a subset of A and Ω is a non-empty finite set of n-ary operations (with n positive) defined on A. With each model (A, D, Ω) is associated the algebra (A, Ω), and (A, Ω) is called a *full algebra* if and only if (iff) for each $\omega \epsilon \Omega$ the range of ω is A itself. A model (A, D, Ω) is called *full* iff its associated algebra is full. The usual two-valued truth-table for the classical propositional calculus is full. An example of a truth-table which is not full is:

C	0	1	2
0	1	1	1
*1	0	1	0
*2	1	1	1

The starred elements are designated. A model $\mathfrak{M}'$ is called a *super model* of a model $\mathfrak{M}$ iff every well-formed formula (wff) valid in $\mathfrak{M}$ is also valid in $\mathfrak{M}'$. Two models are called *equivalent* iff each is a super model of the other.

A propositional calculus P can sometimes be extended to another propositional calculus P' in such a manner that both P and P' have the same class of full models. An example is the pure implicational fragment $S4_I$ of Lewis's S4 and the intuitionist implicational calculus H_I. Also the intuitionist propositional calculus H, the classical propositional calculus K and all intermediate calculi have the same class of full models in which modus ponens is valid. A study of such extensions is made in [4] and [5], and in such a study it is sometimes important to know whether or not an arbitrary finite model has an equivalent full model. This problem is fully solved here for finite models with one operation and is partly solved in the general case. We include applications of our results to some well-known propositional calculi.

For the purpose of this paper we take a fixed but arbitrary language L_0 of order zero with the propositional variables $p_1, p_2, p_3, \ldots$, a non-empty

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